Asset Pricing with Return Asymmetries:
Theory and Tests[*]

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Abstract

I derive an equilibrium asset pricing model incorporating both systematic and idiosyncratic return asymmetries, and show their respective impact on expected returns. With systematic return asymmetry, investors allocate their wealth between the risk-free security, the market portfolio, and a factor which overweights assets with high systematic asymmetry. Investors who prefer positive asymmetry remain underdiversified from a mean-variance perspective to preserve skewness in their portfolio, and idiosyncratic asymmetry therefore is priced in equilibrium. I find that a systematic asymmetry factor and a factor capturing idiosyncratic asymmetry help explain the cross-sectional variation of expected returns across U.S. equities, international equity markets, government bonds, currencies, and commodities. My results offer a risk-based explanation of expected returns that contributes to our understanding of asset pricing across multiple markets.

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1 Introduction

Most investors prefer portfolios with higher positive asymmetry, all else equal, because they occasionally pay large and positive returns. Similarly, they prefer to avoid portfolios with negative asymmetry because they sometimes fall drastically in value. Not all kinds of return asymmetry are equal however; assets with positive *systematic* asymmetry are desirable because they offer the potential for high returns during bad times, while *idiosyncratic* asymmetry is unrelated to periods of high aggregate marginal utility. When the market portfolio is negatively skewed, which indicates that it offers large negative returns more often than large positive returns, investors may prefer to hold assets with high idiosyncratic asymmetry because they cannot obtain positive skewness in their portfolio by buying the market index. This stands in contrast with the irrelevance of idiosyncratic variance in the Capital Asset Pricing Model (CAPM) with unconstrained borrowing. In that model, investors can reach a desired level of portfolio variance simply by adjusting their allocation to the mean-variance efficient market portfolio.

Hence, both systematic and idiosyncratic asymmetries affect equilibrium asset prices and I provide a model which shows how each source of risk is priced. I find that both kinds of return asymmetry help capture the variations in expected returns in equities, government bonds, currencies, and commodities.

When investors have preferences over the asymmetry of their portfolio returns, Harvey and Siddique (2000), Kraus and Litzenberger (1976) and Rubinstein (1973) extend the CAPM and show that expected returns include a compensation for negative systematic coskewness, which measures the contribution of an asset to the skewness of the market portfolio. The assumption on investor preferences in their asset pricing model implies that investors hold well-diversified portfolios.

However, Mitton and Vorkink (2007) show that skewness-loving investors will choose portfolios concentrated in a few stocks to keep diversification from eliminating positive asymmetry. Consequently, they remain exposed to idiosyncratic skewness, and this source of risk is priced in equilibrium. Unfortunately, they do not obtain closed-form results, and therefore it is difficult to disentangle the role of systematic and idiosyncratic asymmetries. I derive in this paper a new asset pricing model in which investors may hold underdiversified portfolios, and characterize the respective equilibrium prices of risk for systematic and idiosyncratic return asymmetries in closed

form.

I make three main contributions. The first contribution is a novel equilibrium asset pricing model that holds when returns display systematic asymmetry. In equilibrium, the expected return for an asset varies with its covariance with the market portfolio and the parameter that governs the skewness of its return distribution. The premium for systematic asymmetry risk depends on whether the market portfolio is negatively or positively skewed. In a negatively skewed market, assets with *positive* asymmetry are valuable because they diversify systematic asymmetry shocks and they increase the skewness of a portfolio. These assets therefore earn lower average returns. In a positively skewed market, assets with *negative* systematic asymmetry are desirable because of their diversification benefit, and therefore have lower expected returns.

I specify a pure endowment economy in which expected utility maximizers invest in one period and consume their wealth in the second period. Investors differ by their preferences, but they all prefer higher expected return (non-satiability) and lower variance (risk aversion), and some prefer positive skewness in their portfolio (decreasing absolute risk aversion). Asset returns follow an asymmetric and fat-tailed distribution which nests the symmetric Student t distribution and the normal distribution as special cases.

A three-fund separation theorem is obtained in equilibrium: Investors allocate their wealth across the risk-free security, the value-weighted market portfolio, and a systematic asymmetry portfolio that overweights high asymmetry assets and underweights low asymmetry assets. Investors with above-average preference for asymmetry tilt their portfolio towards this last risk factor, but their positions are exactly offset by positions held by other investors such that the market clears. To obtain an equilibrium risk premium for skewness, Rubinstein (1973) and Kraus and Litzenberger (1976) invoke a two-fund separation theorem which implies that investors' proportional allocation to risky assets are the same as in the market portfolio. Harvey and Siddique (2000) similarly adopt a representative agent framework. My model complements theirs in a distinct way; while their assumptions on utility imply investors hold well-diversified portfolios, I instead impose structure on asset returns and obtain that investors may hold concentrated portfolios depending on their preference for asymmetry.

My second main contribution is to show under which conditions idiosyncratic asymmetry is priced in equilibrium with expected utility maximizers. Idiosyncratic return asymmetry commands

a separate risk premium from systematic asymmetry when some or all of the asymmetry in returns is idiosyncratic and some investors have preference for skewness. Assets with positive idiosyncratic asymmetry are valuable because they increase the asymmetry of a portfolio, and I show that its price of risk is negative. Some investors are willing to accept lower expected returns for the possibility of very large returns. This is consistent with the negative relation between idiosyncratic skewness and stock returns found by Boyer, Mitton, and Vorkink (2010), and also with investors' preference for lottery stocks modeled in Barberis and Huang (2008), Brunnermeier, Gollier, and Parker (2007), and further documented in Bali, Cakici, and Whitelaw (2011). However, with positive idiosyncratic skewness comes idiosyncratic variance. Since it is not possible to obtain one without the other and investors are risk averse, expected returns also contain a positive risk premium for idiosyncratic variance.

My final contribution is to show empirically the importance of systematic and idiosyncratic asymmetry across different asset classes. I consider U.S. equity portfolios sorted by industry or by size and momentum, developed equity markets sorted by country or by momentum, U.S. Treasury portfolios sorted by maturity or momentum, currency forward portfolios sorted by forward discount or momentum, and commodity futures portfolios sorted by basis or momentum. Average returns for these assets are related to both measures of return asymmetry, especially for portfolios sorted on momentum. To distinguish between their respective role, I run time series regressions using the market portfolio, the systematic asymmetry factor, and a factor that captures idiosyncratic asymmetry. Then, I test for the null hypotheses of zero pricing errors.

Several important findings arise from this empirical investigation. First, consistent with the theory, risk premiums on the systematic asymmetry factors are significant in all asset classes, and positively related to the asymmetry of their respective market portfolio. Further, the systematic asymmetry factors for equities, currencies and commodities, for which the market portfolio is negatively skewed, have offered high returns during prolonged recessions. Investors have been willing to earn lower average returns on this factor in exchange for high and positive returns during bad times. Next, a model that combines the market portfolio and the systematic asymmetry factor leads to important reductions in test statistics of the null hypothesis of zero pricing errors compared to the CAPM. The full asset pricing model that includes factors for both systematic and idiosyncratic asymmetries is outperformed only by models which include a factor based on the

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same variable used to sort test portfolios on the left-hand side. Nevertheless, it is only rejected, as are all other models considered, for size and momentum sorted U.S. equity portfolios, and for Treasury portfolios. Overall, these results provide support for the importance of both sources of return asymmetry in asset pricing.

My paper is related to a growing literature exploring pricing factors across multiple markets. Fama and French (1993) and Koijen, Lustig, and Nieuwerburgh (2012a) investigate risk factors common to the U.S. equity and bond markets. Baker and Wurgler (2012) find similar comovements in government bonds, and large and safe stocks. Several recent studies find striking results using risk factors applied to U.S. and international equities, country equity indices, government bonds, currencies, and commodities. Asness, Moskowitz, and Pedersen (2013) construct value and momentum factors for each, Moskowitz, Ooi, and Pedersen (2012) construct a time series momentum factor, Koijen, Moskowitz, Pedersen, and Vrugt (2012b) use a measure of carry yield, Lettau, Maggiori, and Weber (2013) use the downside beta of Ang, Chen, and Xing (2006), while Frazzini and Pedersen (2013) construct a low-minus-high beta factor. This paper provides a risk-based explanation of expected returns using both systematic and idiosyncratic asymmetries, and contributes to our understanding of asset prices in different markets.

This paper is also related to a vast literature on asymmetric dependence measures beyond coskewness, and their relation with risk premia. Longin and Solnik (2001) and Ang and Bekaert (2002) show that international equity markets exhibit significantly higher correlation in bear markets than in bull markets, while Ang and Chen (2002) and Hong, Tu, and Zhou (2007) find similar asymmetric correlations in U.S. equity portfolios. Ang et al. (2006) and Lettau et al. (2013) estimate a significant risk premium for bearing downside risk, which they measure by the covariance between an asset and the market conditional on the market return being below a threshold. My model introduces a new systematic asymmetry factor derived from micro-foundations, and shows that it helps capture cross-sectional variations of expected returns.

The next section presents an asset pricing model that incorporates both systematic and idiosyncratic return asymmetry. Section 3 contains empirical tests of the model, and Section 4 concludes.

2 Asset Pricing with Return Asymmetries

I discuss in the next section two routes that have been used to solve for equilibrium risk premia in a two-period model with expected utility preferences. Under these traditional approaches, it is difficult to handle both systematic and idiosyncratic asymmetries. In Sections 2.4 and 2.6, I provide a model that captures their respective roles.

2.1 Assumptions and notations

There are I investors in the economy each allocating their initial wealth $W_{i,0}$ for one period and consuming it at the end of the next period. Investors allocate their wealth across N risky assets and one risk-free asset which pays an exogenously determined rate of r_f. Investor i chooses his optimal portfolio allocation ω_i to maximize his expected utility U_i of next period wealth $W_{i,1} = W_{i,0}\left(1 + r_f + \omega_i^\top (r - r_f)\right)$ where $\omega_{i,j}$ is the proportion of his wealth invested in asset j, $\omega_i^\top$ is the transpose of the $N \times 1$ vector ω_i, and r is the $N \times 1$ vector of risky asset returns.

Without loss of generality, denote the return on asset j in excess of the risk-free rate r_f by

$$r_j - r_f \;\;=\;\; E\left[r_j - r_f\right] + \epsilon_j \tag{1}$$

where ϵ_j is a random shock with mean zero. The $N \times 1$ vector of innovations ϵ has a finite covariance matrix Σ, and all investors have homogeneous information. In this and following sections, we will make different assumptions on the distribution of ϵ and discuss their impact on the $N \times 1$ vector of equilibrium risk premia $E\left[r - r_f\right]$.

The first order conditions of investor i's portfolio allocation decision are given by

$$E\left[U_i'\left(W_{i,1}\right)\right] E\left[r_j - r_f\right] = -Cov\left(r_j - r_f, U_i'\left(W_{i,1}\right)\right), \quad \text{for all } j = 1, ..., N, \tag{2}$$

where I have used the definition of covariance and U_i' is the first derivative of the utility function U_i. The main difficulty in obtaining an expression for equilibrium risk premia lies in solving for the covariance on the right hand side of (2). For example, the CAPM is obtained by making an assumption either on the form of the utility function U, on the distribution of asset returns, or both (see Cochrane, 2001, Ch. 9). In what follows, I contrast the ability of the first two methods

to handle return asymmetries, and then introduce my model.[1]

2.2 Assumptions on utility functions

The first approach uses a series expansion to express utility as a function of moments of returns. Consider for example a Taylor expansion of utility $U_i(W_{i,1})$ around expected wealth $E[W_{i,1}]$

$$
\begin{aligned}
U_i(W_{i,1}) &= U_i(E[W_{i,1}]) + U_i'(E[W_{i,1}])W_{i,0}\left(r_{W_{i,1}} - E[r_{W_{i,1}}]\right) \\
&\quad + \frac{U_i''(E[W_{i,1}])W_{i,0}^2}{2}\left(r_{W_{i,1}} - E[r_{W_{i,1}}]\right)^2 + \ldots
\end{aligned}
\tag{3}
$$

where $r_{W_i} = \omega_i^\top(r - r_f)$ is the excess return on investor i's wealth, and U_i'' denotes the second derivative of U_i.

Using a series representation for U_i has two advantages. First, the covariance in investor i's first order condition for asset j becomes a manageable linear combination of different return covariances. To see this, derive the expansion (3) and substitute into the first order condition (2) as

$$
\begin{aligned}
E[r_j - r_f] &= \frac{-U_i''(E[W_{i,1}])W_{i,0}}{E[U_i'(W_{i,1})]}Cov\left(r_j, r_{W_{i,1}} - E[r_{W_{i,1}}]\right) \\
&\quad - \frac{U_i'''(E[W_{i,1}])W_{i,0}^2}{2E[U_i'(W_{i,1})]}Cov\left(r_j, \left(r_{W_{i,1}} - E[r_{W_{i,1}}]\right)^2\right)
\end{aligned}
\tag{4}
$$

which shows that the expected return on asset j required by investor i depends on its covariance with his portfolio's return and squared return[2].

Second, preference theory provides guidance for the sign and intuition of each term multiplying these different covariances. The first term on the right hand side of (4) indicates that a risk averse investor ($U_i'' < 0$) requires a higher risk premium for an asset that co-moves more with his wealth. This is equivalent to a preference for lower portfolio variance as represented by the last term in Equation (3). Also, preference for positive asymmetry $U_i''' > 0$ results in a negative risk premium for assets with positive coskewness[3] $\left(Cov\left(r_j, \left(r_{W_{i,1}} - E[r_{W_{i,1}}]\right)^2\right)\right)$. The condition $U_i''' > 0$ is implied

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[1]The third method imposes more restrictive assumptions than the first two, and I do not discuss it

[2]I cut the Taylor expansion for U_i' in Equation (4) at the second term to focus on return asymmetries. While the standard CAPM stems from a one-term expansion, Kraus and Litzenberger (1976) and Harvey and Siddique (2000) use a two-term expansion to obtain the three-moment CAPM, and Dittmar (2002) analyzes the role of cokurtosis in a model based on a three-term expansion. In a recent contribution, Chabi-Yo (2012) provides a dynamic extension of these models.

[3]Coskewness can alternatively be defined as $Cov\left(r_j, r_m^2\right)$. The difference depends on whether the Taylor expansion

by the desirable property for utility functions of decreasing absolute risk aversion (see Arditti, 1967). Coskewness is related to asset j's contribution to the skewness of investor i's wealth, and is therefore desirable.

To investigate the impact of systematic and idiosyncratic asymmetry in equilibrium, we need to move from investor i's portfolio optimality condition in Equation (4) to a market equilibrium, which requires summing first order conditions across all investors. Unfortunately, the addition of coskewness terms does not produce a risk measure that is independent of each investor's allocation.

This problem can be solved by imposing further assumptions on utility: Rubinstein (1973) and Kraus and Litzenberger (1976) assume utility functions that display linear risk tolerance with equal cautiousness across investors (see Cass and Stiglitz, 1970), and Harvey and Siddique (2000) adopt a representative agent framework. In each case, a two-fund separation theorem holds, all investors' proportional allocations to risky assets are those of the value-weighted market portfolio, and the resulting priced risk measures are the covariance and coskewness with the market portfolio.

However, these assumptions on utility precludes idiosyncratic asymmetry from playing a role. In an economy with heterogeneity in skewness preference, Mitton and Vorkink (2007) show that some investors will remain underdiversified in equilibrium to preserve skewness in their portfolios. As a result they care about the level of idiosyncratic skewness, and this risk is priced. Unfortunately, their model is not available in closed-form which complicates the analysis of the respective role of systematic and idiosyncratic skewness. I provide in Sections 2.4 and 2.6 a model that avoids the above assumption on utility functions and therefore let both systematic and idiosyncratic asymmetry impact equilibrium expected returns.

2.3   Assumptions on the return distribution

Alternatively, the second method for solving Equation (2) consists in assuming a specific distribution for the return shocks ϵ whose analytical tractability allows to disentangle the covariance on the right hand side. For example, under normally distributed innovations, $\epsilon = z$ where $z \sim N(0, \Sigma)$, we can use Stein's Lemma to express investor i's first order condition as

$$E\left[U_i'\left(W_{i,1}\right)\right] E\left[r_j - r_f\right] = -W_{i,0} E\left[U_i''\left(W_{i,1}\right)\right] Cov(r_j, r_{W_i}).$$

in Equation (3) is taken around current wealth $W_{i,0}$ or around the expectation of next period's wealth $E\left[W_{i,1}\right]$.

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Dividing both sides by $-E\left[U_i''\left(W_{i,1}\right)\right]$ and summing over all I investors leads to the following equilibrium relation

$$E\left[r_j - r_f\right] = W_{m,0}\left(\sum_{i=1}^{I}\theta_i^{-1}\right)^{-1}Cov(r_j, r_m) \tag{5}$$

where $W_{m,0} = \sum_{i=1}^{I}W_{i,0}$ and $r_m = \sum_{i=1}^{I}W_{i,0}r_{W_i}$ are the aggregate wealth and its excess return, and $\theta_i = -\frac{E\left[U_i''\left(W_{i,1}\right)\right]}{E\left[U_i'\left(W_{i,1}\right)\right]}$ is investor i's global absolute risk aversion (see Huang and Litzenberger, 1988, Ch. 4). The equilibrium risk premium for asset j is a linear function of its covariance with the market portfolio, and under standard non-satiation and risk aversion assumptions ($U' > 0$ and $U'' < 0$), the price of covariance risk is positive[4].

The pricing equation (5) is also valid for the market portfolio, and substituting its risk premium into (5) leads to the well-known CAPM

$$E\left[r_j - r_f\right] = \beta_j E\left[r_m - r_f\right] \tag{6}$$

where asset j's risk premium depends on its loading $\beta_j = \frac{Cov(r_j, r_m)}{\sigma_m^2}$ on the market portfolio, and σ_m^2 is the market portfolio variance.

This practice of assuming a return distribution to solve Equation (2) is rarely pursued. To be useful, a distribution needs to be a realistic model for asset returns and offer a manageable form of the covariance in Equation (2). While the normal distribution may fall short on the first count, other members of its family—the class of elliptical distributions—offer more flexibility and retain its analytical tractability[5]. Unfortunately, they do not lead to richer empirical predictions (see Owen and Rabinovitch, 1983).

To see this limitation, consider for example another elliptical distribution, the symmetric Student t distribution (ST), whose stochastic representation $\epsilon = \sqrt{g_m}z$ consists of the product of a normally distributed vector z and the square root of an independent inverse gamma random variable g_m, and where the covariance of z is now $E[g_m]^{-1}\Sigma$ to ensure that the innovation covariance matrix $E\left[\epsilon\epsilon^\top\right] = \Sigma$ is preserved[6]. I use a subscript m to emphasize that g_m is a scalar random shock that impacts all assets in the market. Using a generalization of Stein's Lemma given in

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[4]Equation (5) does require additional technical assumptions on utility functions. Specifically, U_i is assumed to be twice differentiable and integrable ($E\left[U_i'\left(W_{i,1}\right)\right] < \infty$ and $E\left[U_i''\left(W_{i,1}\right)\right] < \infty$) for all $i = 1,...I$.

[5]See Landsman and Neslehova (2008) for a generalization of Stein's Lemma for elliptical distributions.

[6]g_m is distributed as $IG\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$ where ν is a degree-of-freedom parameter, see Appendix A.1 for more details.

Appendix B for ST-distributed returns, we obtain by following the same procedure as above

$$E\left[r_j - r_f\right] = W_{m,0} \left(\sum_{i=1}^{I} \theta_i^{ST-1}\right)^{-1} Cov(r_j, r_m) \tag{7}$$

where $\theta_i^{ST} = -\frac{E^{ST^*}\left[U_i''(W_{i,1})\right]}{E\left[U_i'(W_{i,1})\right]}$ is investor i's adjusted global absolute risk aversion and ST^* denotes a probability measure under which the innovations ϵ have fatter tails than under ST. The pricing equation (7) differs from (5) under normally distributed returns only by this adjustment for tail risk in the global absolute risk aversion measures. However, writing Equation (7) for the market portfolio and substituting back into (7) leads to the same factor representation as in the standard CAPM in (6). Owen and Rabinovitch (1983) show that this is the same regardless of the elliptical distribution used. Hence, using more flexible distributions in the elliptical family offers little more in terms of empirical predictions.

The absence of any risk premium for asymmetry in Equations (5) and (7) does not imply investors are indifferent to asymmetries. Rather, it is due to the symmetry of the return distributions I have assumed. Unfortunately, finding an asymmetric and analytically tractable distribution has proven to be a hard task so far. The next section generalizes the normal and Student t distributions to handle both systematic and idiosyncratic return asymmetries.

2.4   A model with systematic asymmetry

I begin by deriving an equilibrium relation using an asymmetric Student t distribution that captures systematic asymmetry. Then, I introduce an extension of the asymmetric t distribution that also handles idiosyncratic asymmetry. In both cases, I present the return distribution using their stochastic representation, and not their probability distribution function. Stochastic representations give the recipe for a random variable, and are useful to grasp how systematic and idiosyncratic asymmetries are generated.

I let innovations in returns follow the asymmetric Student t distribution (AT) from Demarta and McNeil (2005) which I have parametrized such that their first and second moments are preserved ($E\left[\epsilon\right] = 0$ and $E\left[\epsilon\epsilon^\top\right] = \Sigma$). The stochastic representation of the AT distribution is

$$\epsilon = \lambda(g_m - E\left[g_m\right]) + \sqrt{g_m}z, \quad z \sim N\left(0, E\left[g_m\right]^{-1}\left(\Sigma - \sigma_{g_m}^2 \lambda\lambda^\top\right)\right) \tag{8}$$

where λ is a $N \times 1$ vector of asymmetry parameters and $\sigma^2_{g_m}$ is the variance of g_m (see Appendix A.1). Even though the AT is not an elliptical distribution, the symmetric t distribution is nested when $\lambda_j = 0$ for all $j = 1, ..., N$, and the normal distribution is retrieved when $\lambda_j = 0$ for all $j = 1, ..., N$ and the degree-of-freedom of g_m goes to infinity.

The positively skewed shock g_m impacts asset returns in two different ways. First, it scales their overall variance via $\sqrt{g_m}z$. This effect creates non-normal tail risk symmetrically for negative and positive returns. Second, asset j loads on this common shock's deviation from its mean, $\lambda_j (g_m - E[g_m])$. This interaction creates systematic asymmetry in asset returns, the sign of which depends on whether λ_j is positive or negative. When g_m is above its mean, all returns with negative loadings, $\lambda_j < 0$, receive a negative shock. Since inverse gamma random variables are always positively skewed, it creates negative systematic asymmetry for these assets.

Appendix 1 provides an extension of Stein's Lemma for AT-distributed returns. Summing investors' portfolio optimality conditions leads to the main result below which I call the Asymmetric CAPM (A-CAPM).

Proposition 1 (A-CAPM). *In equilibrium with AT-distributed returns, security j requires two risk premia*

$$E\left[r_j - r_f\right] \quad = \quad \gamma Cov\left(r_j, r_m\right) + \left(E\left[r_m - r_f\right] - \gamma \sigma^2_m\right) \frac{\lambda_j}{\lambda_m} \tag{9}$$

where $\lambda_m = \omega_m^\top \lambda$ is the aggregate wealth's asymmetry risk loading, and

$$\gamma = W_{m,0} \left(\sum_{i=1}^{I} \theta_i^{AT-1} \right)^{-1}$$

is an aggregate measure of risk aversion where

$$\theta_i^{AT} = -\frac{E^{AT^*}\left[U_i''(W_{i,1})\right]}{E\left[U_i'(W_{i,1})\right]}$$

is an adjusted measure of global absolute risk aversion for investor i, and AT^* denotes a probability measure whose only difference with the true probability measure is that the systematic asymmetry shock g_m is distributed as $IG\left(\frac{\nu-2}{2}, \frac{\nu}{2}\right)$ instead of $IG\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$.

Proof. See Appendix B. □

The second term on the right hand side of Equation (9) indicates that the premium for systematic asymmetry depends on the sign of the market portfolio's asymmetry. Consider first the case when the market is negatively skewed ($\lambda_m < 0$). To obtain positive asymmetry by investing in the market portfolio, one would have to take a short position and therefore lose the market risk premium. Assets with positive systematic asymmetry $\lambda_j > 0$ are valuable not only because they have high returns during bad times when the market experiences negative asymmetry shocks, but also because they offer positive asymmetry in an environment where the market is negatively skewed. This is a key aspect of how asymmetry differs from variance in a CAPM setting with unconstrained borrowing in which a target variance can be reached by investing only in the market portfolio.

Consider next the case when the market portfolio is positively skewed ($\lambda_m > 0$). An investor can lever up or down the market portfolio to obtain the desired level of positive asymmetry in his portfolio. Assets with lower λs are now valuable from a diversification perspective; they attenuate the impact of below average systematic shock ($g_m - E[g_m] < 0$). In this context, the economic intuition for λ is the same as for β in the CAPM.

In both cases, assets with lower ratios $\frac{\lambda}{\lambda_m}$ are desirable and this source of risk commands a positive risk premium $E\left[r_m - r_f\right] - \gamma\sigma_m^2$. The price premium for positive λ assets in a negative λ_m market decreases when the market Sharpe ratio increases; investors with preference for positive asymmetry are less willing to pay a higher price for positively skewed assets as the market becomes more appealing from a mean-variance perspective.

The implications of Proposition 1 from the perspective of a positive theory of asset pricing are the same as in the model of Kraus and Litzenberger (1976). What have we gained then by assuming the *AT* distribution? I have derived an equilibrium relation without initially invoking a separation theorem, and my model therefore complements those of Rubinstein (1973) and Kraus and Litzenberger (1976). Further, the next section shows that a three-fund separation theorem is obtained, which is a clear distinction from the two-fund separation theorem relied upon in their models. Also, the *AT* distribution provides a stepping stone for a model with idiosyncratic asymmetry which I introduce in Section 2.6.

2.5 Portfolio implications under systematic asymmetry risk

We now examine the portfolio implications by substituting the equilibrium risk premia in Equation (9) into agent i's first order condition in Equation (2).

Corollary 1 (Three Fund Separation). *In an equilibrium under AT-distributed returns, each investor allocates his wealth to the risk-free security, the value-weighted market portfolio ω_m, and a systematic asymmetry portfolio with weights equal to*

$$\omega_{SA} = \left(\frac{1}{1 - \sigma_{g_m}^2 \lambda^\top \Sigma^{-1} \lambda}\right) \Sigma^{-1} \lambda. \tag{10}$$

Agent i allocates a fraction $f_{m,i} = \frac{\gamma}{W_{i,0}\theta_{i,t}}$ of his wealth to the market portfolio and a fraction $f_{SA,i}$ to the systematic asymmetry portfolio, where $f_{SA,i}$ is given in Appendix C.

The asymmetry portfolio $r_{SA} = \omega_{SA}^\top(r - r_f)$ overweights assets with high systematic asymmetry and underweights assets with low asymmetry. Investors with strong preference for asymmetry will tilt their portfolio towards the systematic asymmetry factor, but their positions are exactly offset by investors with weaker or no preference for asymmetry. The rest of their wealth is split between the risk-free security and the value-weighted market portfolio. Summing each investor's positions, we verify that the market for risky asset clears ($\sum_{i=1}^I W_{i,0} f_{m,i} = W_{m,0}$ and $\sum_{i=1}^I W_{i,0} f_{SA,i} = 0$).

By construction, all return asymmetry in the A-CAPM is systematic. The next section generalizes the A-CAPM by introducing idiosyncratic asymmetry.

2.6 A model with systematic and idiosyncratic asymmetries

The asymmetry of an asset under the AT distribution is created by a common shock g_m and is therefore entirely systematic by design. This is an important limitation. For example, the asymmetry of a portfolio with weight ω in this framework is the weighted average of each asset's loading, $\omega^\top \lambda$. Yet, individual assets can exhibit positive asymmetry while aggregating to a negatively skewed market, an empirical observation documented by Albuquerque (2012) and others in equities. Consequently, I introduce in this section the generalized asymmetric Student t distribution (GAT). Compared to the stochastic representation for the AT distribution in Equation 11, there

is one additional term, $\varphi_j(g_j - E[g_j])$, which captures idiosyncratic return asymmetry:

$$\epsilon_j = \varphi_j(g_j - E[g_j]) + \lambda_j(g_m - E[g_m]) + \sqrt{g_m}z_j, \quad \text{for all } j = 1, ..., N \tag{11}$$

where φ_j is a scalar idiosyncratic asymmetry parameter and g_j is an inverse gamma variable independent of g_m and z, and with the same degree-of-freedom as g_m. Therefore, g_j is genuinely an idiosyncratic shock, and g_j and g_m share the same moments (e.g. $E[g_j] = E[g_m]$). As a result, asset j's idiosyncratic asymmetry is uniquely determined by its loading φ_j on g_j.

The asset-specific shock $\varphi_j(g_j - E[g_j])$ adds flexibility by allowing for positive asymmetry in asset j (i.e. $\varphi_j + \lambda_j > 0$), and negative asymmetry in the market portfolio ($\lambda_m = \omega_m^\top \lambda < 0$) in which idiosyncratic shocks have been averaged out. Finally, the covariance of z, given in Appendix A.2, is specified such that the covariance of the innovations ϵ is still Σ.

The GAT is a weak assumption on the distribution of returns. Not only does the GAT distribution nest the AT distribution (when all $\varphi_j = 0$), the symmetric Student t distribution (when all $\varphi_j = 0$ and $\lambda_j = 0$) and the normal distribution (when all $\varphi_j = 0$, $\lambda_j = 0$, and $\nu \to \infty$), it captures prevalent characteristics of financial asset returns such as asymmetric correlation (Longin and Solnik, 2001; Ang and Chen, 2002) and downside beta (Ang et al., 2006; Lettau et al., 2013). Figure 3 in Appendix A.3 and the discussion therein illustrates how the AT distribution captures stylized facts of financial asset returns.

With GAT-distributed returns, Proposition 1 is supplemented with two additional risk premia as follows, and I denote this model as the Generalized Asymmetric CAPM (GA-CAPM).

Proposition 2 (GA-CAPM). *In equilibrium with GAT-distributed returns, security j requires four risk premia*

$$E[r_j - r_f] = \gamma Cov(r_j, r_m) + \pi^{SA}\frac{\lambda_j}{\lambda_m} + \pi^{IV}\varphi_j^2 + \pi^{IA}\varphi_j^3 \tag{12}$$

where π^{SA} is the price of systematic asymmetry risk, π^{IV} is the price of idiosyncratic variance risk, and π^{IA} is the price of idiosyncratic asymmetry risk, all of which are given by the following

expressions:

$$\begin{aligned}
\pi^{SA} &= E\left[r_m - r_f\right] - \gamma\sigma_m^2 + \gamma\sigma_g^2 \sum_{k=1}^{N} \omega_{m,k}^2 \varphi_k^2 + \mathcal{A}^{SA}, \\
\pi^{IV} &= \mathcal{A}_j^{IV} \sigma_g^2, \\
\pi^{IA} &= -\mathcal{A}_j^{IA} \kappa_g,
\end{aligned} \tag{13}$$

where σ_g^2 and κ_g are the asymmetry shocks' variance and third central moment, and $\mathcal{A}^{SA}$, $\mathcal{A}_j^{IV}$ and $\mathcal{A}_j^{IA}$ are constants, all of which depend on investor preferences and are given in Equations (34) in Appendix B. The sign of $\mathcal{A}_j^{IV}$ and $\mathcal{A}_j^{IA}$ are discussed below.

Finally, the measures of aggregate global relative risk aversion contains adjustments for higher order risk

$$\gamma = W_{m,0} \left(\sum_{i=1}^{I} \theta_i^{GAT-1} \right)^{-1}$$

with

$$\theta_i^{GAT} = -\frac{E^{GAT^*}\left[U_i''(W_i)\right]}{E\left[U_i'(W_i)\right]}$$

where GAT^* denotes a probability measure whose only difference with the true probability measure is that the systematic asymmetry shock g_m is distributed as $IG\left(\frac{\nu-2}{2}, \frac{\nu}{2}\right)$ instead of $IG\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$.

Proof. See Appendix B. $\qquad\qquad\square$

Idiosyncratic asymmetry risk impacts expected returns through two different channels which are distinct from the way systematic asymmetry affects expected returns. To see clearly how idiosyncratic asymmetry is compensated in equilibrium, set all systematic asymmetries in Equation (12) to zero ($\lambda_j = 0$ for all j):

$$E\left[r_j - r_f\right] = \gamma Cov\left(r_j, r_m\right) + \mathcal{A}_j^{IV}\sigma_g^2\varphi_j^2 - \mathcal{A}_j^{IA}\kappa_g\varphi_j^3. \tag{14}$$

I show in Appendix B that the constant $\mathcal{A}_j^{IA}$ is always positive under decreasing absolute risk aversion ($U''' > 0$). Given that the third central moment κ_g of an inverse gamma variable is always positive, the risk premium for idiosyncratic asymmetry risk φ_j is negative. Therefore, the GA-CAPM shows that under expected utility preferences, idiosyncratic asymmetry is priced if returns

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display idiosyncratic asymmetry and some investors have a preference for positive skewness.

The second risk premium in Equation (14), $\mathcal{A}_j^{IV} \sigma_g^2 \varphi_j^2$, reflects the tradeoff between idiosyncratic skewness and variance: A skewness-loving investor is willing to pay a price premium for a positively skewed asset, but buying this asset increases the overall variance of his portfolio. The constant $\mathcal{A}_j^{IV}$ contains two terms. The first is a positive constant reflecting the compensation for the increase in idiosyncratic variance caused by investing in a positive φ asset. The second term is negative and cancels the presence of idiosyncratic variance in the covariance risk premium. The overall impact is a positive risk premium for idiosyncratic variance, and a risk premium for covariance only indicative of comovements between an asset and the market.

In the presence of idiosyncratic asymmetry risk, investors still allocate to the risk-free security, the market portfolio, and the systematic asymmetry factor although its weights now contain an adjustment for idiosyncratic variance (see Appendix C). But these three portfolios do not constitute their entire portfolio. The rest of their allocation is tilted towards high idiosyncratic asymmetry assets. Unfortunately, this part of their allocation cannot be separated from each investor's preferences (the exact allocations are given in Appendix C). In the next section, I will combine the market and systematic asymmetry factors proposed in Corollary 1 to a factor that captures idiosyncratic asymmetry, and examine empirically the respective role of each kind of return asymmetry across different asset classes.

3 Empirical Results

In this section, I test for the importance of the systematic asymmetry factor and idiosyncratic asymmetry in different asset classes. My empirical methodology follows three steps:

1. First, I construct time series of monthly returns for test portfolios in four different asset classes: equities (U.S. equity portfolios sorted by industry or by size and momentum, and developed market equity indices sorted by country or by momentum), U.S. Treasury bonds sorted by maturity or momentum, currency forward portfolios sorted by forward discount or momentum, and commodity futures portfolios sorted by basis or momentum (Section 3.1).

2. For each set of test portfolios, I build the systematic asymmetry risk factor from Corollary 1, and a risk factor that captures idiosyncratic asymmetry (Section 3.2).

3. Using the market portfolio and these two asymmetry factors, I run time series regressions and test for the joint hypothesis that all pricing errors are equal to zero (Section 3.3).

3.1 Test portfolios

I provide here a brief description of the test portfolios in each asset class, and provide more details on their source and construction methodology in Appendix D. All returns are monthly, end in December 2012, are denominated in U.S. dollars, and are in excess of the one-month U.S. Treasury bill rate. Tables 1 to 4 present summary statistics for each test portfolio in all asset classes, including their respective starting date.

In what follows, I consider two broad categories of test portfolios. First, I use portfolios sorted by a variable specific to each asset class. U.S. equity portfolios are sorted by industry (see Lewellen, Nagel, and Shanken, 2010), developed equity markets by country, U.S. Treasury bonds by maturity, currency forwards by forward discount (Lustig, Roussanov, and Verdelhan, 2011), and commodity futures by basis (Yang, 2013; Gorton, Hayashi, and Rouwenhorst, 2012). Then, I consider portfolios sorted by momentum in each asset class. In all cases below, momentum refers to the one year lagged return skipping the last month (using returns from month $t-12$ to $t-2$). Asness et al. (2013) provide compelling evidence on the profitability of momentum strategies in different asset classes, and I show that there is a strong relation in all cases between momentum and return asymmetries.

For U.S. equity, I use 10 value-weighted portfolios sorted by industry and 25 sorted by size and momentum. Returns start in July 1961 for a total number of $T = 618$ observations. Longer time series are available, but I set the starting point to coincide with the starting date of U.S. Treasuries to facilitate the comparison between stocks and government bonds. The market portfolio is the CRSP value-weighted portfolio of all stocks.

Panels A in Tables 1 and 3 report for these portfolios their annualized average return and volatility, β with the market portfolio, skewness, and coskewness with the market portfolio. In the last row of each panel, I report the cross-sectional correlation between each measure and sample average excess returns. These test portfolios are challenges for empirical tests of the CAPM, as shown by the negative correlation between β and average returns for size and momentum portfolios, and near zero correlation for industry portfolios.

I test for the null hypothesis of zero skewness and zero non-normal coskewness using boot-

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strapped samples under the null hypothesis of normal returns[7]. For both sets of portfolios, the null hypothesis of zero coskewness is rejected everywhere at the 1% confidence level. Further, the cross-sectional correlation between average returns and coskewness is negative in both panels, and negative for skewness for size and momentum portfolios. These negative correlations are consistent with investors requiring compensation for negative return asymmetries. The negative correlations for coskewness in both sets of portfolios are in line with its importance in asset pricing as shown by Harvey and Siddique (2000).

Next, panel B of Table 1 contains summary statistics for 16 value-weighted developed country equity markets. Returns are from Datastream, and start in February 1973 ($T = 479$). Using each index's market capitalization in U.S. dollars, I build a value-weighted market portfolio containing all 16 markets. Rouwenhorst (1998) and Asness et al. (2013) find strong evidence of profitability of momentum strategies in international equity markets, and I also build five value-weighted quintile portfolios of these 16 markets sorted each month by momentum. Summary statistics for these portfolios are found in panel B of Table 3. The first column in panel B confirms that momentum is profitable in international equity markets, with the winner quintile earning on average $6.91\% - 0.71\% = 6.20\%$ annually more than the bottom quintile. The null hypotheses of zero coskewness are rejected everywhere. Similar to U.S. equity portfolios, coskewness is strongly negatively related to average returns, and so is skewness for momentum sorted portfolios.

U.S. Treasury fixed term bond indices are available from CRSP for maturities of 1, 2, 5, 7, 10, 20, and 30 years starting in July 1961 ($T = 618$). These test portfolios are strictly speaking not portfolios as a single representative bond is chosen each month. Because of the limited number of different assets, I build three equally weighted portfolios using each month the 30th and 70th percentiles of momentum. I also construct a value-weighted market portfolio of all Treasury issues available from CRSP, see Appendix D for further details.

Panel A in Table 2 reports on constant maturity indices. There is a strong and positive correlation between βs and average returns, and the t statistics of the βs (not reported) indicate that they are all significant. This suggests that Treasury βs, using a value-weighted Treasury market portfolio, may play a more important role in pricing these assets than the equity market does for equity portfolios. The skewness and coskewness are significantly positive everywhere except for

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[7]The coskewness of a bivariate normal distribution is $Cov(r, r_m^2) = 2E[r_m - r_f] Cov(r, r_m)$.

skewness for 5-year bonds.

The correlation of average returns with skewness is negative, but positive for coskewness. This is exactly in line with the economic implications of the GA-CAPM. Treasuries are the only asset class for which the value-weighted portfolio of all securities is positively skewed. In such market, assets with high systematic asymmetry are not desired for their asymmetry because positive portfolio skewness can be obtained by buying the market index. Instead, lower systematic asymmetry is valuable for diversification purposes. Hence, higher coskewness implies higher average returns as found in the data.

The average excess returns of momentum sorted portfolios, reported in panel A of Table 4, indicate that momentum is profitable in U.S. Treasuries with the top portfolio earning 1.14% more than the bottom portfolio. I consider momentum-sorted portfolios in U.S. Treasuries to have a comprehensive analysis of the relation between momentum profitability and return asymmetries across all asset classes. However, given the small number of portfolios, I am careful not to reach any firm conclusions on momentum in this asset class.

Next, panel B in Table 2 contains summary statistics for portfolios of one-month forwards for 16 developed country exchange rates. Lustig et al. (2011), Lettau et al. (2013), and Koijen et al. (2012b) all report important data problems using a wider sample of currencies (failures of covered interest rate parity and missing data). I follow Lustig et al. (2011) and consider a smaller dataset of developed countries, which has the advantage of being comparable to our sample of international equity market indices. Spot and one-month forward rates are from Datastream, and the sample starts in November 1983 ($T = 350$). Following Lettau et al. (2013), I use the value-weighted U.S. equity market as the market portfolio.

Based on Lustig et al. (2011), I build five equally weighted portfolios of long one-month forward positions sorted by their average forward discount during the previous month. The forward discount $ln\left(\frac{S_t}{F_t}\right)$, where S_t is the spot rate and F_t is the one-month forward rate (both denominated in U.S. dollar per unit of foreign currency), is indicative of the interest rate differential between the foreign country and the U.S..

Portfolios are sorted from low to high interest rate differential with the top quintile portfolio outperforming the bottom by a spread of $5.78\% - 0.43\% = 5.35\%$ annually. Average returns are positively related to market β, and negatively to coskewness. This is consistent with Lettau

et al. (2013) who find that a model with β and downside β, defined as the slope coefficient conditional on low market returns, explain the cross-sectional variations in expected returns. The skewness for the low interest rate differential portfolio is 0.31, and decreases monotonically to a value of -0.30 for the top portfolio. This negative relation between average returns and skewness is in line with the findings of Brunnermeier, Nagel, and Pedersen (2008).

Currency forward portfolios sorted by momentum, as reported in panel B of Table 4, produce a similar spread in average returns between the winner and the loser portfolios of $4.72\% - 1.67\% = 3.05\%$. Correlations between averages returns and volatility, β, and skewness are negative. Currencies are the only case in which momentum sorted portfolios do not produce a negative correlation between average returns and coskewness.

Finally, I construct five basis and five momentum sorted commodity futures portfolios. The universe consists of the 24 commodities found in the Goldman Sachs Commodity Index. This index selects commodity futures based on liquidity, and covers six categories (energy, industrial metals, grains, soft agriculture products, livestock products and precious metals). The sample starts in January 1970 ($T = 338$), and as for currency forward portfolios, I use the U.S. equity value-weighted index as the market portfolio.

Each month I sort available commodity futures into five equally weighted portfolios. The sort is based on their previous month basis, which is defined as $ln\left(\frac{F_t^{T_0}}{F_t^{T_{12}}}\right)/(T_{12} - T_0)$ where $F_t^{T_{12}}$ is the futures price with the maturity T_{12} closest to one year ahead and $F_t^{T_0}$ is the futures price with maturity T_0 nearest, but later than the end of next month. Each portfolio return is the equally weighted return on the futures with maturity T_0.

Annual average returns for high basis and high momentum portfolios are respectively more than 12% and 15% annually, and decreases monotonically down to the lowest basis and momentum portfolios. The spread in basis-sorted portfolios is consistent with the results of Yang (2013) and Gorton et al. (2012), although my sample composition and length are different from theirs. Coskewness is significantly negative and skewness is positive in both sets of portfolios.

By definition, an idiosyncratic shock affects a single asset. Using portfolios, in which idiosyncratic shocks tend to average out, is likely to understate the importance of idiosyncratic asymmetry. This is an important concern for our equity portfolios because they contain a large number of stocks. On the other hand, bond fixed term indices are built using only one representative issue at a time,

and the small number of currencies and commodities results in concentrated quintile portfolios. I compare in this paper the impact of return asymmetries for representative assets across different asset classes. Interesting insights could be obtained from an analysis at the security level, but I leave this for future work. The next section uses these test portfolios to construct return asymmetry factors.

3.2 Building the risk factors

In this section, I first build the systematic risk factor, and then construct a risk factor that captures idiosyncratic asymmetry. These factors along with the market portfolio are used in the next section in time series regressions to explain expected returns for different assets.

My model shows that investors should allocate their wealth between the risk-free security, the value-weighted market portfolio, a well-defined systematic asymmetry portfolio, and the rest of their allocation based on the idiosyncratic asymmetry of each asset. While the systematic asymmetry portfolio is uniquely determined, how the rest of their allocation is related to idiosyncratic asymmetry depends on each investors' preferences. I begin by showing how to construct the systematic asymmetry factor, and then discuss how I capture empirically idiosyncratic asymmetry.

One possibility is to estimate the parameters of the GAT distribution and form the systematic asymmetry factor. Instead, I propose below an estimator which uses only sample covariances and coskewness to build the systematic asymmetry factor from the A-CAPM, and then use residual skewness to build an idiosyncratic asymmetry factor. Observe first that the weights of the systematic asymmetry factor r_{SA} in Corollary 1 are proportional to the vector of asymmetry parameters λ standardized by the covariance matrix Σ:

$$\omega_{SA} = \left(\frac{1}{1 - \sigma_{g_m}^2 \lambda^\top \Sigma^{-1} \lambda} \right) \Sigma^{-1} \lambda \propto \Sigma^{-1} \lambda. \tag{15}$$

Using again the analytical convenience of the GAT distribution, Appendix A.2 shows that these asymmetry parameters are proportional to a function of their covariance and coskewness as

$$\lambda \propto Cos(r, r_m) - 2 \frac{\sigma_g^2}{E[g]} \lambda_m Cov(r, r_m) \tag{16}$$

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where $Cos(r, r_m) = E\left[(r_j - E[r_j])(r_m - E[r_m))^2\right]$ denotes the $N \times 1$ vector of coskewness terms. Substituting (16) into (15) leads to

$$\omega_{SA} \propto \Sigma^{-1} Cos(r, r_m) - 2\frac{\sigma_g^2}{E[g]}\lambda_m \omega_m. \qquad (17)$$

Since the second term is just the market factor scaled by a constant, we can test the A-CAPM by regressing asset returns on a constant, the market portfolio, and an approximated systematic asymmetry factor:

$$\tilde{r}_{SA} = \tilde{\omega}_{SA}^\top (r - r_f), \text{ where } \tilde{\omega}_{SA} = k\Sigma^{-1} Cos(r, r_m). \qquad (18)$$

For simplicity, I scale the weights using the scalar k such that the realized (ex post) volatility of this risk factor is equal to the market's realized volatility over the whole sample.

By construction, any asymmetry left after accounting for systematic risk is idiosyncratic. For each test portfolio, I regress its excess returns on a constant, its market portfolio, and its systematic asymmetry factor $\tilde{r}_{SA}$. Following Asness et al. (2013) and Frazzini and Pedersen (2013), I form a long-short portfolios r_{IA} containing all portfolios within a set of test portfolios by weighting them according their residual skewness' rank.

More formally, let z_j be the rank of test portfolio j's residual skewness. I set its weight $\omega_{IA,j} = z_j - \bar{z}$ where $\bar{z}$ is the average rank. Finally, I scale all weights such that the realized (ex post) volatility of the idiosyncratic asymmetry factor $r_{IA} = \omega_{IA}^\top (r - r_f)$ is the same as the market portfolio's volatility over the whole sample.

The GA-CAPM contains one risk premium for idiosyncratic asymmetry and one for idiosyncratic variance. By constructing a factor based on residual skewness, I implicitly control for idiosyncratic variance since skewness is the average third power of residuals standardized by their standard deviation. Also, the risk premium for idiosyncratic asymmetry in the GA-CAPM relates to φ^3, and not skewness. However, there is a one-to-one relation between the two measures (see Appendix A.2), and using skewness instead of an estimate of φ does not impact r_{IA} because using one or the other does not affect ranks.

Tables 5 and 6 present summary statistics for all market portfolios, systematic asymmetry

and idiosyncratic asymmetry factors. Each panel reports the average excess return, volatility, and skewness for these factors, as well as their cross-correlations. In all but one case, the risk premium on the systematic asymmetry factor is significant. More importantly, its average excess return, as predicted by the GA-CAPM, is strongly and positively related to the skewness of the market portfolio. For example, it is negative for equities, currency forwards and commodity futures, but positive for Treasury bonds for which the value-weighted market portfolio is rightly skewed.

Further economic insights are provided in Figures 1 and 2 which show the cumulative returns of market portfolios and systematic asymmetry factors against the NBER recession dates defined by the NBER's Business Cycle Dating committee. In negatively skewed markets, I report the cumulative return of a short position in the systematic asymmetry factor to facilitate comparison with the market index. Tables 5 and 6 show that the market factor and the systematic asymmetry factors are less than perfectly correlated. Nevertheless, the systematic asymmetry factors in equities, currencies and commodities tend to experience positive returns (the short positions in these factors depicted in the figures fall) especially when bad times occur and the market portfolios lose in value. This pattern is not observed for Treasuries whose market index is positively skewed. In the next section, I formally test for the importance of return asymmetries for explaining cross-sectional variations in expected returns.

3.3 Time series regressions

In this section, I test my asset pricing model using the market portfolio and the two risk factors constructed in the previous section. The empirical methodology closely follows Fama and French (2012). For each group of test portfolios, I estimate the following time series regression using monthly returns

$$r_{t,j} - r_{t,f} = \alpha_j + F_t \beta_j + u_{t,j}, \quad t = (1, ..., T) \tag{19}$$

for each asset j. In this regression, α_j is the average return for asset j not explained by the risk factors, F_t is the vector of risk factors at time t, β_j is asset j's loadings on these factors, and $u_{t,j}$ is an error term. I then test for the joint hypothesis that all pricing errors α_j are equal to zero.

I report regression results in Table 7 for portfolios sorted by asset class specific variables, and in Table 8 for momentum sorted portfolios. I report the F-test statistic of Gibbons, Ross, and

Shanken (1989) (GRS) and its p-value, and the χ^2 test statistic and its p-value which I estimate by GMM using a Newey-West estimator with 3 lags for the moment conditions' covariance matrix. Both GRS and χ^2 tests assess the null hypothesis that all pricing errors are equal to zero, but they differ on the underlying statistical assumptions (see Cochrane, 2001, Ch. 12 for further details). I also report the average absolute pricing error $|\alpha_j|$ expressed in basis points per month, the average standard error of the intercept coefficients expressed in basis points per month, and the average R^2 across all regressions in Equation (19).

As benchmarks, I use the single factor CAPM, followed by other benchmarks specific to each asset class. U.S. and international equity portfolios are tested using the Fama-French three-factor model, in which the small-minus-big market capitalization factor for size and the high-minus-low book-to-market ratio factor for value are added to the market portfolio. This model is then augmented with the winner-minus-loser (previous 12-month return) factor (Carhart, 1997). These models are referred to as FF3 and FF3+Momentum. I use factors constructed with U.S. stocks because longer time series are available. For example, Fama and French (2012) construct size, value and momentum factors with international stocks, but these factors start in 1990. For Treasury bonds, I compute a slope factor which is the difference between the monthly return on the 10-year Treasury bond minus the monthly return on the 1-year Treasury bond (see for example Fama and French, 1993). For currency forwards, I build a high-minus-low rank-based forward discount factor (HML_{FX}) which is long the higher quintile portfolios sorted on forward discount and short the lower quintile portfolios (see Lustig et al., 2011). Based on their ranks, the five quintile portfolios sorted from low to high forward discount have weights of $-\frac{2}{3}$, $-\frac{1}{3}$, 0, $\frac{1}{3}$, and $\frac{2}{3}$. Similarly, I build a high-minus-low basis commodity factor (HML_C) following Yang (2013). I refer to these models respectively as CAPM+TERM, CAPM+HML_{FX}, and CAPM+HML_C.

The asymmetry factors $\tilde{r}_{SA}$ and r_{IA} are then added sequentially to the market portfolio r_m to examine their individual impact, and the last line of each panel reports on the GA-CAPM which includes the market portfolio and both asymmetry factors.

Consider first the results from GRS tests. First, the A-CAPM, in which the market factor is augmented with the systematic asymmetry factor, leads to important reductions in test statistics compared to the CAPM in all cases, except for momentum sorted currency forward portfolios for which the reduction is marginal. Second, in five out of 10 sets of portfolios, the GA-CAPM model

has the smallest test statistic of all models considered. In other cases, it still compares favorably to asset-class specific benchmarks. Not surprisingly, the GA-CAPM is outperformed in some cases when the benchmark model contains a factor built from the same variable used to sort the portfolios on the left-hand side of the regressions. For instance, the CAPM+HML$_{FX}$ model has a test statistic value of 0.55 for currency forward portfolios sorted on forward discount versus a value of 0.72 for the GA-CAPM. The same result holds for basis-sorted commodity futures portfolios (1.37 versus 2.63) and U.S. equity size and momentum portfolios (3.70 versus 4.71). The two other cases where the GA-CAPM does not produce the lowest test statistics are for the Treasury bond market, but its underperformance is marginal (7.72 versus 7.61 for maturity sorted portfolios, and 7.75 versus 7.26 for momentum sorted portfolios). Third, the GA-CAPM survives the GRS test at the 1% level everywhere except for size and momentum sorted U.S. equity portfolios and all Treasury portfolios. However, all models are rejected for these portfolios. Finally, note that the above conclusions are also reached using the χ^2 test.

Lower test statistics are partially explained by lower average pricing errors everywhere except for currency forward portfolios, and by more precisely estimated pricing errors in all cases (see columns titled average $|\alpha|$ and average σ_α). All models produce high average R^2 for all equity and Treasury portfolios. However, the increases in R^2 are modest for currency forward portfolios (0.04 to 0.19) even though the models are not rejected by the GRS and χ^2 tests. Interestingly, while the CAPM+HML$_C$ model results in a lower test statistic than the GA-CAPM for commodity portfolios sorted on basis, only the latter brings a large increase in explanatory power. Specifically, the CAPM and CAPM+HML$_C$ both have low R^2s (0.03 and 0.20), while the GA-CAPM yields an R^2 of 0.55. Overall, the results in Tables 7 and 8 provide support for the importance of systematic and idiosyncratic return asymmetry in these asset classes.

4 Conclusion

This paper derives an asset pricing model that incorporates both systematic and idiosyncratic return asymmetries. When returns display systematic asymmetry, investors allocate their wealth between the risk-free security, the value-weighted market portfolio, and a systematic asymmetry portfolio tilted towards rightly skewed assets. While idiosyncratic return asymmetry commands a

negative risk premium, the compensation for systematic asymmetry depends on whether the market portfolio is negatively or positively skewed. Assets with positive systematic asymmetry are valuable in a negatively skewed market because they increase the asymmetry of a portfolio. If returns on the market portfolio are rightly skewed however, assets with negative systematic asymmetry are valuable because they diversify systematic asymmetry shocks.

The empirical results provide support for my asset pricing model. First, realized average returns on the systematic asymmetry factors are significant and positively related to the asymmetry of the market portfolio, as predicted by theory. Second, time series regression results show that the GA-CAPM, in which the market portfolio is augmented with a systematic asymmetry and an idiosyncratic asymmetry factor, performs favorably in U.S. equities, in international equity markets, in U.S. Treasury bonds, in currency forwards, and in commodity futures.

Systematic and idiosyncratic return asymmetry are hard to measure and to distinguish. More sophisticated estimation methods using higher frequency returns could be used to further study their dynamics in each of these asset classes. Also, I have analyzed return asymmetries on portfolios. It may prove interesting in future work to examine idiosyncratic asymmetry risk at the security level where its relative importance is likely to be larger.

A   Return Distributions

A.1   Moments of inverse gamma variables

The first three central moments of an inverse gamma distributed variable g with parameter ν, $g \sim IG\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$, are given by

$$
\begin{aligned}
E[g] &= \frac{\nu}{\nu - 2}, \\
\sigma_g^2 &= E[(g - E[g])^2] = \frac{2\nu^2}{(\nu - 4)(\nu - 2)^2}, \\
\kappa_g &= E[(g - E[g])^3] = \frac{16\nu^3}{(\nu - 6)(\nu - 4)(\nu - 2)^3}.
\end{aligned}
$$

A.2   The generalized asymmetric t distribution

The stochastic representation of the $N \times 1$ vector $r - r_f$ under GAT-distributed returns is

$$
r - r_f = E\left[r - r_f\right] + \varphi \circ (g - E[g]) + \lambda\left(g_m - E[g]\right) + \sqrt{g_m}z \tag{20}
$$

where $E\left[r - r_f\right]$, φ, and λ are $N \times 1$ vectors of mean, idiosyncratic asymmetry, and systematic asymmetry parameters, $\circ$ denotes the Hadamard (element-by-element) product, g is a $N \times 1$ vector of independent inverse gamma variables, g_m is a inverse gamma variable, $z \sim N\left(0, E[g]^{-1}\left(\Sigma - \sigma_g^2\left(\lambda\lambda^\top + diag(\varphi)^2\right)\right)\right)$ where $diag(\varphi)^2$ is a N by N matrix with φ_j^2 as diagonal elements and zero everywhere else, and g, g_m and z are independent of each other. For simplicity, all inverse gamma variables have the same degree-of-freedom, $g, g_m \sim IG\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$, and hence I refer to their moments using only subscript g.

The AT distribution is obtained by setting all idiosyncratic asymmetry parameters to 0 ($\varphi_j = 0$ for all $j = 1, ..., N$). If all systematic asymmetry parameters λ_j are also equal to zero, we get the symmetric t distribution. Further, if the degree-of-freedom goes to infinity, then the normal distribution is obtained.

From the stochastic representation in (20), one can verify that

$$
E[\varphi \circ (g - E[g]) + \lambda\left(g_m - E[g]\right) + \sqrt{g_m}z] = 0,
$$

and

$$Cov(r - r_f) = \Sigma.$$

The skewness of asset j is given by

$$E\left[\left(\frac{r_j - E\left[r_j\right]}{\sigma_j}\right)^3\right] = \frac{3\sigma_g^2}{E[g]}\left(\frac{\lambda_j}{\sigma_j}\right) + \left(\kappa_g - 3\frac{\sigma_g^4}{E[g]}\right)\left(\frac{\lambda_j}{\sigma_j}\right)^3 + \kappa_g\left(\frac{\varphi_j}{\sigma_j}\right)^3 - 3\frac{\sigma_g^4}{E[g]}\left(\frac{\varphi_j^2\lambda_j}{\sigma_j^3}\right) \tag{21}$$

where $\sigma_j = \sqrt{\Sigma_{jj}}$ is the volatility of asset j. Finally, the coskewness of asset j with the market is

$$E\left[(r_j - E\left[r_j - r_f\right])(r_m - E\left[r_m - r_f\right])^2\right] = \left(\kappa_g - 3\frac{\sigma_g^4}{E[g]}\right)\lambda_m^2\lambda_j + \frac{\sigma_g^2}{E[g]}\sigma_m^2\lambda_j$$

$$+2\frac{\sigma_g^2}{E[g]}\lambda_m Cov(r_j, r_m). \tag{22}$$

Isolating λ_j in the above equation, we can see that it is proportional to

$$\lambda_j \propto E\left[(r_j - E\left[r_j - r_f\right])(r_m - E\left[r_m - r_f\right])^2\right] - 2\frac{\sigma_g^2}{E[g]}\lambda_m Cov(r_j, r_m). \tag{23}$$

This result is used to derive a moment estimator for the weights ω_{SA} of the systematic asymmetry factor.

When all idiosyncratic asymmetry loadings equal zero, $\varphi_j = 0$ for all j, we obtain the asymmetric t distribution (Demarta and McNeil, 2005) with likelihood function

$$\mathcal{L}\left(\Theta; r - r_f | \varphi = 0\right) = \frac{2^{1 - \frac{(\nu + N)}{2}} K_{\frac{\nu + N}{2}}\left(\Psi\left(\Theta\right)\right) e^{(r - E[r] + E[g]\lambda)^\top \Omega(\Theta)^{-1}\lambda}}{\Gamma\left(\frac{\nu}{2}\right)(\pi\nu)^{\frac{N}{2}}|\Omega\left(\Theta\right)|^{\frac{1}{2}}\left(\Psi\left(\Theta\right)\right)^{-\frac{\nu + N}{2}}\left(1 + \frac{\Delta(\Theta)}{\nu}\right)^{\frac{\nu + N}{2}}} \tag{24}$$

where

$$\Omega\left(\Theta\right) = E[g]^{-1}\left(\Sigma - \sigma_g^2\left(\lambda\lambda^\top + diag(\varphi)^2\right)\right),$$

$$\Delta\left(\Theta\right) = (r - E\left[r\right] + E[g]\lambda)^\top \Omega\left(\Theta\right)^{-1}(r - E\left[r\right] + E[g]\lambda),$$

$$\Psi\left(\Theta\right) = \sqrt{(\nu + \Delta\left(\Theta\right))\lambda^\top \Omega\left(\Theta\right)^{-1}\lambda},$$

and $K(\cdot)$ is the modified Bessel function of the third kind. Unfortunately, the likelihood function when $\varphi \neq 0$ is unknown. However, it can be obtained by convolution:

$$\mathcal{L}\left(\Theta; r - r_f\right) = \int_{(0,\infty)^N} \mathcal{L}\left(\Theta; r - r_f - \varphi \circ g | \varphi = 0\right) \prod_{j=1}^{N} \frac{\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} g_j^{-\frac{\nu}{2}-1} e^{-\frac{\nu}{2g_j}}}{\Gamma\left(\frac{\nu}{2}\right)} dg \qquad (25)$$

where the last term is the density function of all independent idiosyncratic shocks g_j.

A.3   Examples of GAT-distributed returns

The GAT distribution is flexible enough to capture higher order risk, including tail risk and multivariate asymmetries. I graph in Figure 3 the contour lines of its probability density function given in Equation (25), and its implied asymmetric correlation and downside beta for four different cases involving a hypothetical market return r_m and an asset return r_j. The top left graph in panel A shows the contour lines of the probability density function when both the market and asset j exhibit negative systematic asymmetry ($\lambda_m = \lambda_j = -0.1$), and no idiosyncratic asymmetry ($\varphi_m = \varphi_j = 0$). In all panels, I set the expected returns to zero, the volatilities to 20%, the linear correlation to 0.5, and the market idiosyncratic asymmetry parameter to zero.

The univariate skewness in both assets is apparent from the shape of the density function in the lower left quadrant. The second graph below in panel A reports the threshold correlation and downside beta. For each negative (positive) return threshold on the horizontal axis, the threshold correlation reported on the left vertical axis is the linear correlation computed on the subset of returns in which both returns are below (above) the threshold. For each negative (positive) return threshold, the downside beta reported on the right vertical axis is the ratio of the covariance to the market variance computed on the subset of returns in which the market return is below (above) this threshold. It is a defining characteristic of the normal distribution that the threshold correlation goes to zero as the return threshold gets further from zero, unless the linear correlation is 1 or -1. The positive level of threshold correlation is therefore indicative of its ability to capture non-normal tail risk. Also, the GAT distribution models the stylized fact that dependence is higher for negative returns, as illustrated by the asymmetric pattern both in threshold correlation and downside beta.

I consider in panel B the case when asset j has positive idiosyncratic asymmetry ($\varphi_j = 0.15$). The higher asymmetry for asset j is visible in the upper right quadrant of the density function, and

in the threshold correlation for positive returns. Increasing the idiosyncratic asymmetry of asset j increases its univariate skewness, but has little effect on both downside dependence measures, as shown in the left part of the lower graph in panel B. This mechanism can explain how individual stocks can have positive skewness, while the index displays negative skewness.

Panels C and D present the same two cases, but with positive systematic asymmetry for asset j. Positive systematic asymmetry has a strong impact on downside β which shows higher values for positive return thresholds than for negative.

B   Proof of Proposition 2

The proof, which relies on the generalized Stein's Lemma below, is given in the general case of GAT-distributed returns. The proof of the pricing equations for ST-distributed in Equation (7) and AT-distributed returns in Equation (9) as well as their respective Stein's Lemma can be obtained by applying the appropriate parameter restrictions. In the case of the AT distribution, all idiosyncratic asymmetry parameters are equal to zero ($\varphi_j = 0$ for all $j = 1, ..., N$), whereas all asymmetry parameters are equal to zero in the ST distribution ($\varphi_j = \lambda_j = 0$ for all $j = 1, ..., N$).

We first need the following lemma, which uses Stein's lemma.

Lemma 1 (Generalized Stein's Lemma). *Let the N-vector X be distributed from a GAT distribution with parameters $E[r - r_f]$, Σ, ν, λ, and φ, and let $h(X)$ be a differentiable $\mathbb{R}^N \mapsto \mathbb{R}$ function such that $\frac{\partial h(X)}{\partial X_i}$ exists almost everywhere and $E\left[\frac{\partial}{\partial X_i} h(X)\right] < \infty$ for $i \in (1, ..., N)$. Then,*

$$
\begin{aligned}
Cov\left(X, h(X)\right) &= \Sigma E^{GAT^*}\left[\nabla h(X)\right] - \sigma_g^2 \left(\lambda \lambda^\top + diag(\varphi)^2\right) E^{GAT^*}\left[\nabla h(X)\right] \\
&\quad + E\left[h(X)\left(g_m - E[g]\right)\right]\lambda + E\left[h(X)\varphi \circ (g - E[g])\right]
\end{aligned}
$$

where $\nabla h(X) = \left(\frac{\partial h(X)}{\partial X_1}, \ldots, \frac{\partial h(X)}{\partial X_N}\right)^\top$ is the gradient vector of function $h(X)$, and E^{GAT^*} indicates that the expected value is taken under a different probability measure. Under GAT^*, X has the same stochastic representation as a GAT-distributed variable, except for $g_m \sim IG\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$ which is now $g_m^* \sim IG\left(\frac{\nu-2}{2}, \frac{\nu}{2}\right)$[8]. Note that X is no longer a GAT random variable under GAT^*.

---

[8]All the moments of g_m^* are larger than the moments of g_m

$$
E[g_m^{*k}] = \frac{\left(\frac{\nu}{2}\right)^k}{\prod_{l=1}^k \left(\frac{\nu}{2} - 1 - l\right)} > E[g_m^k] = \frac{\left(\frac{\nu}{2}\right)^k}{\prod_{l=1}^k \left(\frac{\nu}{2} - l\right)}.
$$

Proof. Conditional on g_m and g, X is normally distributed with mean vector $E[r - r_f] + \lambda(g_m - E[g]) + \varphi \circ (g - E[g])$ and covariance matrix $\frac{g_m}{E[g]}(\Sigma - \sigma_g^2(\lambda\lambda^\top + diag(\varphi)^2))$. We obtain by conditioning on the inverse gamma variable g_m and g

$$
\begin{aligned}
Cov(X, h(X)) &= E[(X - E[r - r_f])h(X)] \\
&= E[E[(X - E[r - r_f] - \lambda(g_m - E[g]) - \varphi \circ (g - E[g]))h(X) \mid g_m, g]] \\
&\quad + E[\lambda(g_m - E[g])h(X)] + E[\varphi \circ (g - E[g])h(X)] \\
&= \left(\Sigma - \sigma_g^2(\lambda\lambda^\top + diag(\varphi)^2)\right)E\left[\nabla h(X)\frac{g_m}{E[g]}\right] \\
&\quad + E[\lambda(g_m - E[g])h(X)] + E[\varphi \circ (g - E[g])h(X)] \quad\quad (26)
\end{aligned}
$$

where we have used Stein's lemma for normally distributed random vectors. Now, notice that

$$
\begin{aligned}
E\left[h(X)\frac{g_m}{E[g]}\right] &= \int_{\mathbb{R}^N}\int_{(0,\infty)^{N+1}} h\left(E[r - r_f] + \lambda(g_m - E[g]) + \varphi \circ (g - E[g]) + \sqrt{g_m}z\right) \\
&\quad \times \left(\frac{g_m}{E[g]}\right)\frac{\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} g_m^{-\frac{\nu}{2}-1} e^{-\frac{\nu}{2g_m}}}{\Gamma\left(\frac{\nu}{2}\right)} \prod_{j=1}^N \frac{\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} g_j^{-\frac{\nu}{2}-1} e^{-\frac{\nu}{2g_j}}}{\Gamma\left(\frac{\nu}{2}\right)} \\
&\quad \times \phi_N\left(z; \mathbf{0}_N, \frac{g_m}{E[g]}\left(\Sigma - \sigma_g^2(\lambda\lambda^\top + diag(\varphi)^2)\right)\right)dg_m dg dz \\
&= \int_{\mathbb{R}^N}\int_{(0,\infty)^{N+1}} h\left(E[r - r_f] + \lambda(g_m - E[g]) + \varphi \circ (g - E[g]) + \sqrt{g_m}z\right) \\
&\quad \times \frac{\left(\frac{\nu}{2}\right)^{\frac{\nu-2}{2}} g_m^{-\frac{\nu-2}{2}-1} e^{-\frac{\nu}{2g_m}}}{\Gamma\left(\frac{\nu-2}{2}\right)} \prod_{j=1}^N \frac{\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} g_j^{-\frac{\nu}{2}-1} e^{-\frac{\nu}{2g_j}}}{\Gamma\left(\frac{\nu}{2}\right)} \\
&\quad \times \phi_N\left(z; \mathbf{0}_N, \frac{g_m}{E[g]}\left(\Sigma - \sigma_g^2(\lambda\lambda^\top + diag(\varphi)^2)\right)\right)dg_m dg dz \\
&= E^{GAT^*}[h(X)]
\end{aligned}
$$

where ϕ_N is the multivariate normal density. Substituting into equation (26) and rearranging produces the desired results. $\qquad\square$

Let's now consider the N-vector of first order conditions of agent i

$$
E[U_i'(W_{i,1})]E[r - r_f] = -Cov(r - r_f, U_i'(W_{i,1})).
$$

Replacing the covariance by its exact expression in Lemma 1, and writing the last two terms as covariances yields

$$
\begin{aligned}
E\left[U_i'\left(W_{i,1}\right)\right]E\left[r - r_f\right] = & -W_{i,0}E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]\Sigma\omega_i \\
& +W_{i,0}\sigma_g^2 E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]\left(\lambda\lambda^\top + diag(\varphi)^2\right)\omega_i \\
& -Cov\left(U_i'\left(W_{i,1}\right), g_m\right)\lambda - Cov\left(U_i'\left(W_{i,1}\right), \varphi \circ g\right).
\end{aligned}
\tag{27}
$$

Dividing both sides by $-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]$ and summing over all I agents gives

$$
\begin{aligned}
E\left[r - r_f\right] = & W_m\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1}\Sigma\omega_m - W_{m,0}\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1}\sigma_g^2\lambda_m\lambda - W_{m,0}\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1}\sigma_g^2\varphi_m^2 \\
& -\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1}Cov\left(\sum_{i=1}^I \frac{U_i'\left(W_{i,1}\right)}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}, g_m\right)\lambda \\
& -\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1}Cov\left(\sum_{i=1}^I \frac{U_i'\left(W_{i,1}\right)}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}, \varphi \circ g\right)
\end{aligned}
\tag{28}
$$

where I have defined a global measure of risk aversion $\theta_i = -\frac{E^{GAT^*}\left[U_i''(W_{i,1})\right]}{E\left[U_i'(W_{i,1})\right]}$ (see Huang and Litzenberger, 1988, Ch. 4), and where φ_m^2 is a vector with typical element $\omega_{m,j}\varphi_j^2$. Equation (28) is valid for all securities, including the market portfolio r_m, which leads to

$$
\begin{aligned}
E\left[r_m - r_f\right] = & W_m\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1}\sigma_m^2 - W_{m,0}\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1}\sigma_g^2\lambda_m^2 - W_{m,0}\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1}\sigma_g^2\omega_m^\top\varphi_m^2 \\
& -\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1}Cov\left(\sum_{i=1}^I \frac{U_i'\left(W_{i,1}\right)}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}, g_m\right)\lambda_m \\
& -\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1}Cov\left(\sum_{i=1}^I \frac{U_i'\left(W_{i,1}\right)}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}, \omega_m^\top(\varphi \circ g)\right)
\end{aligned}
\tag{29}
$$

Define $\gamma = W_{m,0}\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1}$ as the aggregate global relative risk aversion, and substitute equation (29) into (28) to get

$$
\begin{aligned}
E\left[r - r_f\right] = & \gamma Cov\left(r, r_m\right) + \left[E\left[r_m - r_f\right] - \gamma\sigma_m^2 + \gamma\sigma_g^2\omega_m^\top\varphi_m^2 + Cov\left(\tilde{\theta}^{-1}, \omega_m^\top(\varphi \circ g)\right)\right]\frac{\lambda}{\lambda_m} \\
& -\gamma\sigma_g^2\varphi_m^2 - Cov\left(\tilde{\theta}^{-1}, \varphi \circ g\right)
\end{aligned}
\tag{30}
$$

where

$$\tilde{\theta}^{-1} = \left(\sum_{i=1}^{I} \frac{E\left[U_i'\left(W_{i,1}\right)\right]}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}\right)^{-1} \sum_{i=1}^{I} \frac{U_i'\left(W_{i,1}\right)}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}.$$

The final step consists in separating the last term in Equation (30) into an idiosyncratic variance and an idiosyncratic asymmetry effect. I use a Taylor expansion of $U_i'\left(W_{i,1}\right)$ around $E[W_{i,1}] = W_{i,0}(1 + r_f + \omega_i^\top E\left[r - r_f\right])$ up to the third order :

$$\begin{aligned}
U_i'\left(W_{i,1}\right) &\approx U_i'\left(E[W_{i,1}]\right) + U_i''\left(E[W_{i,1}]\right) W_{i,0}\omega_i^\top\left(r - E\left[r\right]\right) \\
&\quad + \frac{U_i'''\left(E[W_{i,1}]\right) W_{i,0}^2}{2}\left(\omega_i^\top\left(r - E\left[r\right]\right)\right)^2 \\
&= \mathcal{T}_{0,i} + \mathcal{T}_{1,i}\omega_i^\top\left(r - E\left[r\right]\right) + \mathcal{T}_{2,i}\left(\omega_i^\top\left(r - E\left[r\right]\right)\right)^2
\end{aligned} \tag{31}$$

where $\mathcal{T}_{0,i}$, $\mathcal{T}_{1,i}$, and $\mathcal{T}_{2,i}$ are constants. Substituting the Taylor expansion into the last term in Equation (30), we get

$$\begin{aligned}
-Cov\left(\tilde{\theta}^{-1}, \varphi \circ g\right) &= -\left(\sum_{i=1}^{I} \frac{E\left[U_i'\left(W_{i,1}\right)\right]}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}\right)^{-1} \\
&\quad \times \left[Cov\left(\sum_{i=1}^{I} \frac{\mathcal{T}_{1,i}\omega_i^\top\left(r - E\left[r\right]\right)}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}, \varphi \circ g\right)\right. \\
&\quad \left. + Cov\left(\sum_{i=1}^{I} \frac{\mathcal{T}_{2,i}\left(\omega_i^\top\left(r - E\left[r\right]\right)\right)^2}{-2E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}, \varphi \circ g\right)\right]
\end{aligned} \tag{32}$$

Consider the first covariance inside the brackets for asset j

$$\begin{aligned}
Cov\left(\sum_{i=1}^{I} \frac{\mathcal{T}_{1,i}\omega_i^\top\left(r - E\left[r\right]\right)}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}, \varphi_j g_j\right) &= E\left[\sum_{i=1}^{I} \frac{\mathcal{T}_{1,i}\omega_i^\top\left(r - E\left[r\right]\right)}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}\varphi_j(g_j - E[g])\right] \\
&= \left(\sum_{i=1}^{I} \frac{\mathcal{T}_{1,i}\omega_{i,j}}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}\right) \sigma_g^2 \varphi_j^2
\end{aligned}$$

where the last equality is obtained using the fact that the shock g_j is independent of all other

shocks. Similarly, the second covariance for asset j becomes

$$Cov\left(\sum_{i=1}^{I}\frac{\mathcal{T}_{2,i}\left(\omega_i^{\top}(r-E\left[r\right])\right)^2}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]},\varphi_j g_j\right) = E\left[\sum_{i=1}^{I}\frac{\mathcal{T}_{2,i}\left(\omega_i^{\top}(r-E\left[r\right])\right)^2}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}\varphi_j(g_j-E[g])\right]$$

$$= \left(\sum_{i=1}^{I}\frac{\mathcal{T}_{2,i}\omega_{i,j}^2}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}\right)\kappa_g\varphi_j^3$$

where the sum is always positive under decreasing absolute risk aversion ($U_i''' > 0$). Finally, the pricing equation for asset j is

$$\begin{aligned}
E\left[r_j - r_f\right] &= \gamma Cov\left(r_j, r_m\right) + \left[E\left[r_m - r_f\right] - \gamma\sigma_m^2 + \gamma\sigma_g^2\sum_{k=1}^{N}\omega_{m,k}^2\varphi_k^2 + \mathcal{A}^{SA}\right]\frac{\lambda_j}{\lambda_m} \\
&\quad + \mathcal{A}_j^{IV}\sigma_g^2\varphi_j^2 - \mathcal{A}_j^{IA}\kappa_g\varphi_j^3
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
\mathcal{A}^{SA} &= Cov\left(\tilde{\theta}^{-1}, \omega_m^{\top}(\varphi\circ g)\right), \\
\mathcal{A}_j^{IV} &= \left(\sum_{i=1}^{I}\frac{E\left[U_i'\left(W_{i,1}\right)\right]}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}\right)^{-1}\left(\sum_{i=1}^{I}\left(\frac{-U_i''\left(E[W_{i,1}]\right)}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}\right)W_{i,0}\omega_{i,j}\right) - \gamma\omega_{m,j}, \\
\mathcal{A}_j^{IA} &= \left(\sum_{i=1}^{I}\frac{E\left[U_i'\left(W_{i,1}\right)\right]}{-E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}\right)^{-1}\sum_{i=1}^{I}\left(\frac{U_i'''\left(E[W_{i,1}]\right)}{-2E^{GAT^*}\left[U_i''\left(W_{i,1}\right)\right]}\right)W_{i,0}^2\omega_{i,j}^2.
\end{aligned} \tag{34}$$

C Portfolio Implications

I derive here the portfolio holdings in the GA-CAPM. I consider then the case with no idiosyncratic asymmetry which leads to the three-fund separation obtained in the A-CAPM in Corollary 1.

Substituting the equilibrium risk premia in Equation (28) into investor i's first order condition

in Equation (27), we get an allocation of

$$
\begin{aligned}
\omega_i \;=\;& f_{m,i}\omega_m \\
&+ f_{SA,i}\left(\Sigma - \sigma_g^2\left(\lambda\lambda^\top + diag(\varphi)^2\right)\right)^{-1}\lambda \\
&+ \frac{1}{W_{i,0}}\left(\Sigma - \sigma_g^2\left(\lambda\lambda^\top + diag(\varphi)^2\right)\right)^{-1}\left[Cov\left(\frac{U_i'(W_{i,1})}{-E^{GAT^*}\left[U_i''(W_{i,1})\right]},\varphi\circ g\right)\right. \\
&\left. - \theta_i^{-1}\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1} Cov\left(\sum_{i=1}^I \frac{U_i'(W_{i,1})}{-E^{GAT^*}\left[U_i''(W_{i,1})\right]},\varphi\circ g\right)\right]
\end{aligned}
$$

where

$$
\begin{aligned}
f_{m,i} \;=\;& \frac{\gamma}{\theta_i W_{i,0}}, \\
f_{SA,i} \;=\;& \frac{1}{W_{i,0}}\left[Cov\left(\frac{U_i'(W_{i,1})}{-E^{GAT^*}\left[U_i''(W_{i,1})\right]},g_m\right)\right. \\
&\left. - \theta_i^{-1}\left(\sum_{i=1}^I \theta_i^{-1}\right)^{-1} Cov\left(\sum_{i=1}^I \frac{U_i'(W_{i,1})}{-E^{GAT^*}\left[U_i''(W_{i,1})\right]},g_m\right)\right].
\end{aligned}
$$

Using the same Taylor expansion as in Equation (31), I separate the last term into two portfolios:

$$
\begin{aligned}
\omega_{IV,i} \;=\;& \frac{1}{W_{i,0}}\left(\Sigma - \sigma_g^2\left(\lambda\lambda^\top + diag(\varphi)^2\right)\right)^{-1} \\
&\times\left(\left[\frac{U_i''(E[W_{i,1}])W_{i,0}\omega_{i,j}}{-E^{GAT^*}\left[U_i''(W_{i,1})\right]} - \theta_i^{-1}\left(\sum_{k=1}^I \theta_k^{-1}\right)^{-1}\sum_{k=1}^I \frac{U_k''(E[W_k])e_k\omega_{k,j}}{-E^{GAT^*}\left[U_k''(W_k)\right]}\right]\circ\varphi^2\sigma_g^2\right)
\end{aligned}
$$

where φ^2 is a $N\times 1$ vector having φ_j^2 as elements, and

$$
\begin{aligned}
\omega_{IA,i} \;=\;& \frac{1}{W_{i,0}}\left(\Sigma - \sigma_g^2\left(\lambda\lambda^\top + diag(\varphi)^2\right)\right)^{-1} \\
&\times\left(\left[\frac{U_i'''(E[W_{i,1}])W_{i,0}^2\omega_{i,j}^2}{-2E^{GAT^*}\left[U_i''(W_{i,1})\right]} - \theta_i^{-1}\left(\sum_{k=1}^I \theta_k^{-1}\right)^{-1}\sum_{k=1}^I \frac{U_k'''(E[W_k])e_k^2\omega_{k,j}^2}{-2E^{GAT^*}\left[U_k''(W_k)\right]}\right]\circ\varphi^3\kappa_g^3\right)
\end{aligned}
$$

where φ^3 is a $N\times 1$ vector having φ_j^3 as elements.

Finally, to obtain the three-fund separation theorem in Corollary 1, set all idiosyncratic asymmetry parameters to 0 ($\varphi_j = 0$ for all $j = 1, ..., N$). The weights of the systematic asymmetry factor are then

$$
\begin{aligned}
\omega_{SA} &= \left(\Sigma - \sigma_g^2 \lambda \lambda^\top \right)^{-1} \lambda \\
&= \left(\frac{1}{1 - \sigma_g^2 \lambda^\top \Sigma^{-1} \lambda} \right) \Sigma^{-1} \lambda.
\end{aligned}
$$

where the last equality is obtained using the Sherman-Morrison formula.

D   Data Description

This section describes the sources and construction methodologies of market and test portfolios. The momentum measures used below always refer to the lagged one year return dropping the most recent month (using returns from month $t - 12$ to $t - 2$).

The U.S. equity portfolios and the U.S. equity factors (size, value, and momentum) are obtained from Ken French's website, and the market portfolio is the value-weighted portfolio of all NYSE, AMEX, and NASDAQ stocks. The 25 size and momentum portfolio compositions are formed each month, and they contain all NYSE, AMEX, and NASDAQ stocks for which market equity is available at the end of last month, and returns are available over the past 12 months. The portfolios are the intersection of five size (market capitalization) and five momentum portfolios. Both measures use the quintiles from NYSE stocks.

My sample of developed country equity markets consists of Datastream indices for 16 countries: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Singapore, Switzerland, the United Kingdom, and the United States. I obtain total returns and market values in U.S. dollars starting in January 1973. The value-weighted market portfolio contains all 16 market indices and is formed using the lagged market values. Each month t, I sort countries into five value-weighted portfolios based on momentum quintiles.

Returns for the U.S. Treasury bond portfolios are the monthly holding period returns of the fixed-term indices obtained from CRSP. We use indices for maturities of 1, 2, 5, 7, 10, 20, and 30 years. At the end of each month, a representative bond for each maturity is chosen and held for

the next month. A representative bond is the most recently issued among all that are fully taxable, non-callable, non-flower, and at least 6 months from, but closest to the maturity date. Flower bonds are considered if no bond meets these criteria.

I construct a value-weighted market portfolio for the U.S. Treasury market. For each Treasury issue j with an available total face amount outstanding from CRSP ($TOTOUT_{t,j}$), I compute a return

$$
r_{t,j} = \begin{cases} \frac{0.5(BID_t + ASK_t - BID_{t-1} - ASK_{t-1}) + ACCINT_t - ACCINT_{t-1} + PDINT_t}{0.5(BID_{t-1} + ASK_{t-1}) + ACCINT_{t-1}} & \text{if } ASK_t \text{ is not missing} \\ \frac{BID_t - BID_{t-1} + ACCINT_t - ACCINT_{t-1} + PDINT_t}{BID_{t-1} + ACCINT_{t-1}} & \text{if } ASK_t \text{ is missing} \end{cases}
$$

and an end-of-month price

$$
P_{t,j} = \begin{cases} \frac{BID_t + ASK_t}{2} + ACCINT_t & \text{if } ASK_t \text{ is not missing} \\ BID_t + ACCINT_t & \text{if } ASK_t \text{ is missing} \end{cases}
$$

where BID_t, ASK_t, $ACCINT_t$, and $PDINT_t$ are the bid price, ask price, accrued interest and coupon interest payment at time t. This follows the same procedure as how monthly holding period returns are computed for fixed term indices. Using the total face amount outstanding $TOTOUT_{t,j}$, I get a market value as

$$
MV_{t,j} = P_{t,j} \times \frac{TOTOUT_{t,j}}{100}.
$$

Then, I obtain the return of the value-weighted U.S. Treasury market index by computing

$$
r_{m,t}^{Treasury} = \sum_{j=1}^{N_t} \left(\frac{MV_{t,j}}{\sum_{k=1}^{N_t} MV_{t,k}} \right) r_{t,j}
$$

where N_t is the number of different Treasury issues for month t.

I obtain spot exchange rates and one-month forward rates from Datastream for 16 countries: Belgium, France, Germany, Italy, and Netherlands (all replaced by the Euro starting in January 1999), Australia, Canada, Denmark, Hong Kong, Japan, New Zealand, Norway, Singapore, Sweden, Switzerland, and the United Kingdom. All rates are expressed in U.S. dollar per unit of foreign currency, and are collected by Barclays Bank International. Not all exchange rates are available each month; we have a minimum of seven and a maximum of 15 over time. The introduction

of the euro is responsible for a decrease in the number of currencies available. Following Lettau et al. (2013), I use the value-weighted U.S. equity index as the market portfolio.

My test portfolio construction methodology is inspired by Lustig et al. (2011). For each currency j, I compute the return on a position that goes long forward one unit of foreign currency at the end of month $t-1$, and that sells the foreign currency at the spot rate at the end of month t, creating an excess return for month t of

$$r_{t,j}^{FX} = S_{t,j} - F_{t-1,j}.$$

Then, I compute using daily quotes within month $t-1$ the average forward discount $FD_{t-1,j}$:

$$FD_{t-1,j} = \frac{1}{D_{t-1}} \sum_{k=1}^{D_{t-1}} ln \left(\frac{S_{k,j}^d}{F_{k,j}^d} \right)$$

where F_k^d and S_k^d are the one-month and spot rates on day k in month $t-1$, and D_{t-1} is the number of days in month $t-1$. I use a monthly average of daily forward discounts to prevent end-of-month erroneous quotes. Finally, I sort currencies available in month t into five quintile portfolios based on their previous month average forward discount, and compute the equally weighted average return for each quintile. Using the same methodology, I construct five equally weighted portfolios sorted by momentum.

I use the composition of the Goldman Sachs Commodity Index (GSCI) as my sample of commodity futures. This index contains 24 traded contracts, covering 6 categories: energy (WTI crude oil, Brent crude oil, RBOB gasoline, heating oil, gas oil, and natural gas), industrial metals (aluminum, copper, lead, nickel, and zinc), precious metals (gold and silver), agriculture - grains and oilseeds (wheat, Kansas wheat, corn, and soybeans), agriculture - softs (cotton, sugar, coffee, and cocoa), and livestock (feeder cattle, live cattle, and lean hogs). As for currencies, not all commodities are available over the whole sample. The sample starts with seven commodities, and ultimately reaches 24 in 2005. As for currency forwards, I use the value-weighted U.S. equity index as the market portfolio.

I follow Yang (2013) in constructing five quintile portfolios sorted on basis. First, I obtain future quotes from Bloomberg for all available maturities. For each commodity futures, I construct a time series of total returns by investing each month in the futures contract with maturity T_0, where T_0

is the nearest maturity coming later than the end of the month. I also compute for month t and commodity j a one-year basis $B_{t,j}$ as

$$B_{t,j} = \frac{ln\left(\frac{F_{t,j}^{T_0}}{F_{t,j}^{T_{12}}}\right)}{T_{12} - T_0}$$

where $F_{t,j}^{T_{12}}$ is the price of the futures contract with maturity T_{12}, which is closest to one year ahead, and $F_{t,j}^{T_0}$ is the price of the futures contract with maturity T_0. Each month, I compute the equally weighted average returns of five portfolios sorted by the quintiles of the previous month basis. I also construct five equally weighted portfolios sorted each month by momentum.

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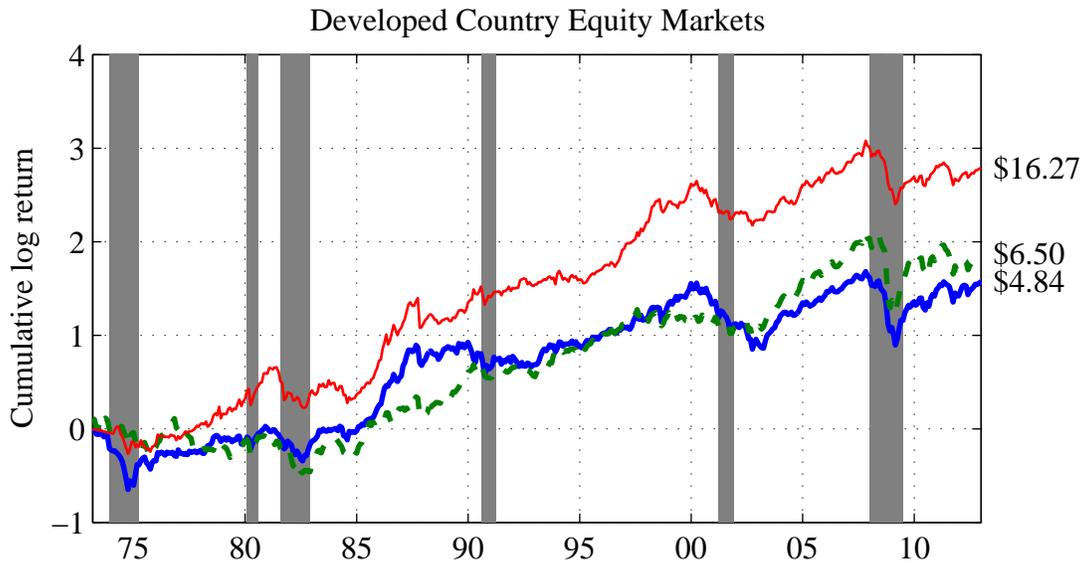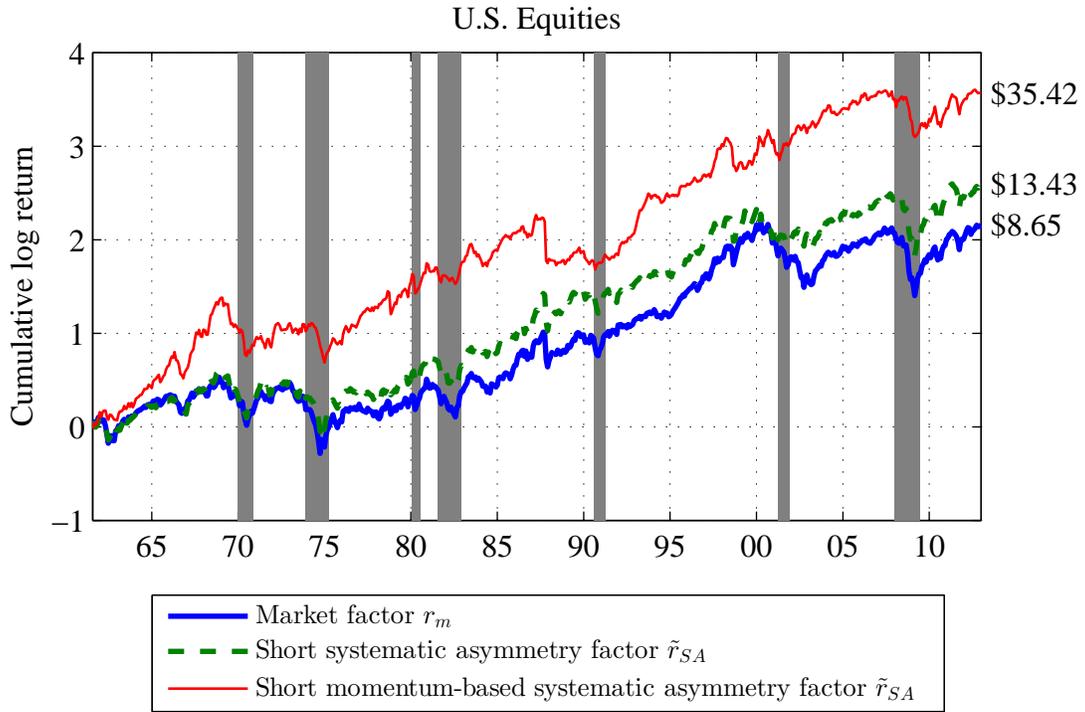
Figure 1 Market Portfolio and Systematic Asymmetry Factors in Equity Markets
I graph the log of cumulative returns of the value-weighted market portfolio, the systematic asymmetry factor based on test portfolios sorted by asset class specific variables, and the systematic asymmetry factor based on test portfolios sorted by momentum. The top graph plots factors for the U.S. equity market, and the bottom reports on developed equity markets. The gray bars indicate official NBER recession dates. Final values of 1$ invested in each factor are reported on the right axis. I show short positions in the systematic asymmetry factors, such that their cumulative returns are comparable to the market portfolio.
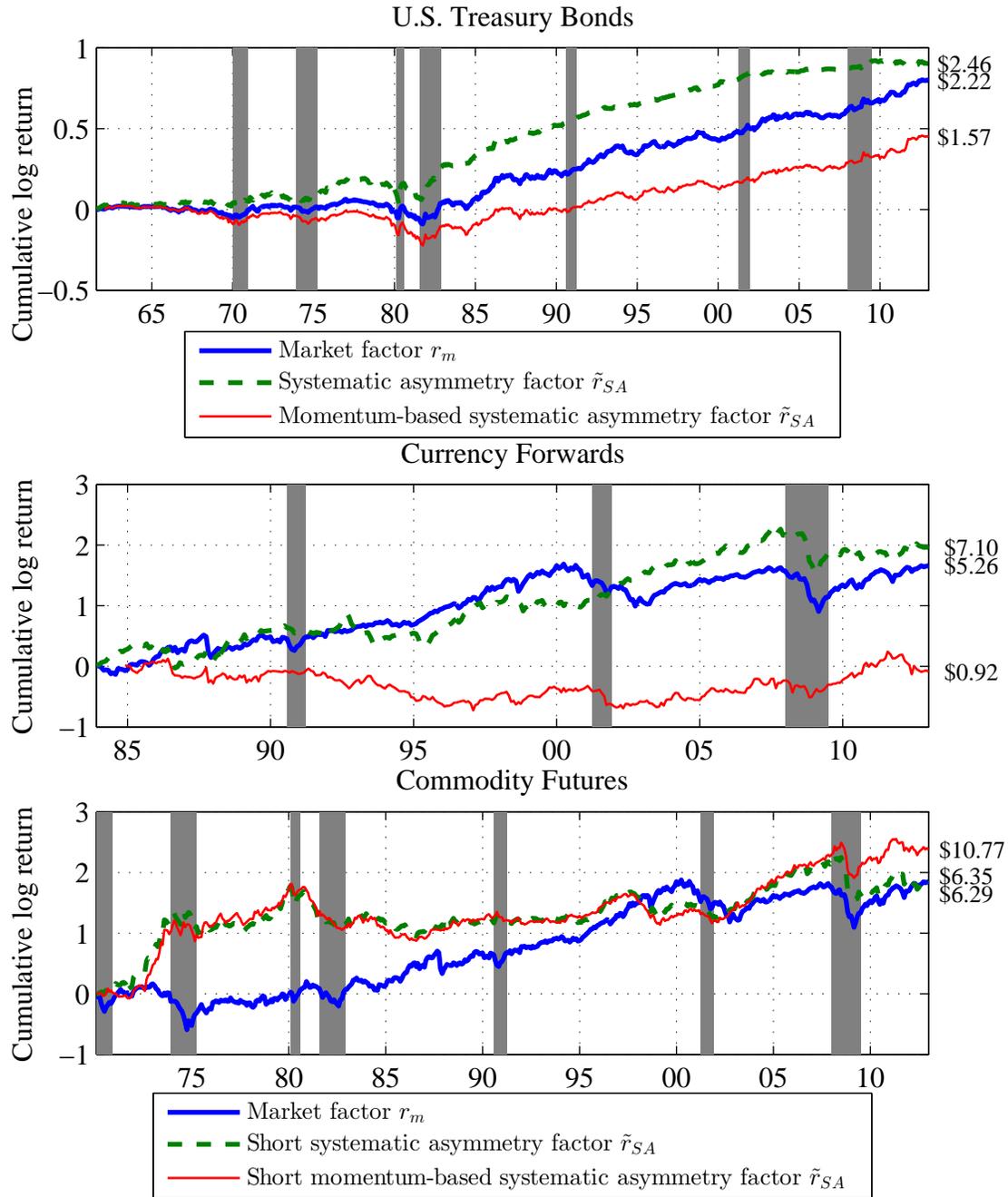
Figure 2 Market Portfolio and Systematic Asymmetry Factors in the Bond, FX, and Commodity Markets

I graph the log of cumulative returns of the value-weighted market portfolio, the systematic asymmetry factor based on test portfolios sorted by asset class specific variables, and the systematic asymmetry factor based on test portfolios sorted by momentum. The top graph plots factors for the U.S. Treasury market, the middle graph for the currency market, and the bottom for the commodity market. The gray bars indicate official NBER recession dates. Final values of 1$ invested in each factor are reported on the right axis. For currency forwards and commodity futures, I show short positions in the systematic asymmetry factors, such that their cumulative returns are comparable to the market portfolio. I graph long positions in the systematic asymmetry factors in the Treasury market which displays positive return asymmetry.
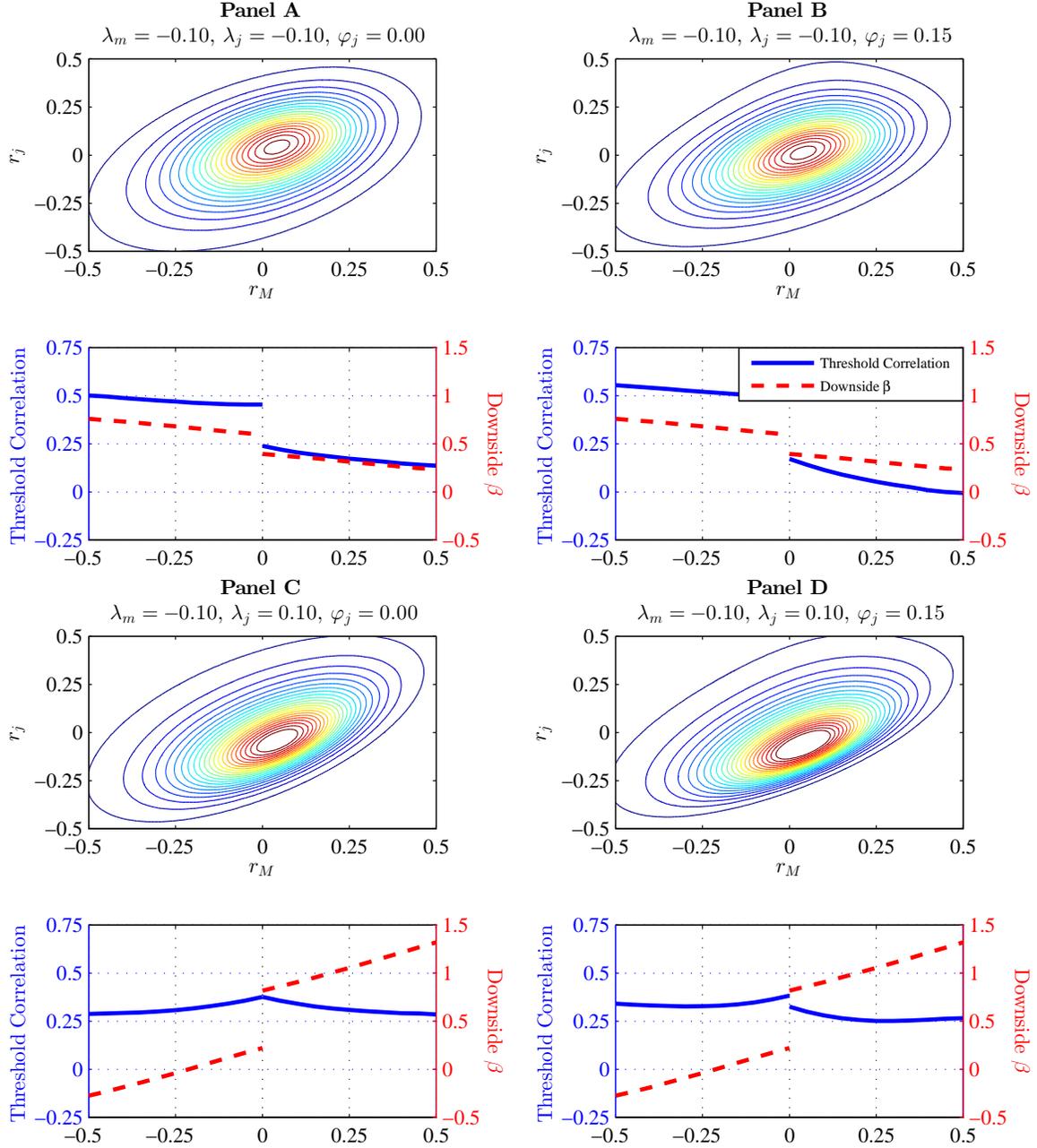
Figure 3 Density Function, Threshold Correlation and Downside Beta of the *GAT* **Distribution**

In each panel, I present the density contour plots for the *GAT* distribution in the top graph, and its threshold correlation (left vertical axis) and downside β functions (right vertical axis) in the bottom graph. The top two panels report on the case with negative systematic asymmetry risk for asset j (λ_j=-0.1), and the bottom two panels on the case with positive loading (λ_j=0.1). The left two panels report on the case with no idiosyncratic asymmetry risk for asset j (φ_j=0), and the right two panels on the case with positive asymmetry (φ_j=0.15). In all panels, the expected return is set to 0, the volatility to 20%, the market systematic asymmetry risk loading to -0.1, the market idiosyncratic asymmetry risk loading to 0, and the correlation to 0.5. For each negative (positive) return threshold on the horizontal axis, the threshold correlation is the linear correlation computed on the subset of returns in which both returns are below (above) the threshold. For each negative (positive) return threshold, the downside beta is the ratio of the covariance to the market variance computed on the subset of returns in which the market return is below (above) this threshold.

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Test Portfolio	Annualized Mean Excess Return(%)	Annualized Volatility (%)	Market β	Skewness	Coskewness $(x10^6)$
Panel A: 10 U.S. Equity Industry Portfolios - 07/1961 to 12/2012					
Consumer Non-Durables	7.37	15.15	0.81	-0.30^{**}	-23.01^{**}
Consumer Durables	5.62	21.95	1.13	0.17	-33.88^{**}
Manufacturing	6.07	17.37	1.04	-0.45^{**}	-29.47^{**}
Energy	8.11	18.68	0.80	-0.01	-25.09^{**}
Business Equipment	6.10	22.93	1.27	-0.20^{*}	-29.86^{**}
Telecommunications	4.97	16.21	0.77	-0.15	-20.65^{**}
Wholesale	7.00	18.23	1.01	-0.27^{**}	-28.76^{**}
Healthcare	7.11	17.31	0.85	0.04	-16.63^{**}
Utilities	4.91	14.02	0.54	-0.11	-6.22^{**}
Other	5.81	18.57	1.10	-0.43^{**}	-29.49^{**}
Correlation with $E[r]$		0.07	0.03	0.06	-0.20
Panel B: 16 Developed Equity Market Portfolios - 02/1973 to 12/2012					
Australia	8.41	25.02	1.05	-0.72^{**}	-55.02^{**}
Austria	6.89	23.42	0.78	0.31^{**}	-55.05^{**}
Belgium	7.17	20.39	0.90	-0.40^{**}	-38.06^{**}
Canada	6.31	19.17	0.94	-0.57^{**}	-38.27^{**}
Denmark	8.48	20.49	0.82	-0.22	-45.68^{**}
France	8.45	23.39	1.10	-0.22	-28.82^{**}
Germany	6.54	20.93	0.97	-0.32^{**}	-42.91^{**}
Hong Kong	11.88	34.06	1.16	0.45^{**}	-59.30^{**}
Ireland	7.94	25.04	1.10	0.22^{*}	-28.45^{**}
Italy	5.72	26.38	0.98	0.12	-33.61^{**}
Japan	3.97	21.29	0.99	0.29^{*}	-11.36^{**}
Netherlands	7.91	19.34	1.03	-0.78^{**}	-45.09^{**}
Singapore	7.56	29.03	1.19	0.57^{**}	-35.27^{**}
Switzerland	7.46	17.88	0.85	-0.36^{**}	-29.62^{**}
United Kingdom	8.01	22.44	1.08	1.06^{**}	-11.47^{**}
United States	5.66	15.79	0.88	-0.42^{**}	-27.15^{**}
Correlation with $E[r]$		0.59	0.42	0.12	-0.55

Table 1 Summary Statistics For Equity Portfolios

I report sample annualized average excess returns, annualized volatility, market β, skewness, and coskewness with the market of monthly excess returns. Panel A reports on 10 industry sorted U.S. equity portfolios, and panel B on 16 developed country equity markets. The market portfolio for U.S. equity portfolios is the CRSP value-weighted market portfolio. The market portfolio for international equity portfolios is the value-weighted portfolio with all country indices. In each panel, I report the cross-sectional correlation of each measure with average excess returns. I compute bootstrap p-values for the null hypotheses of zero skewness and zero non-normal coskewness. I simulate 10,000 bivariate normal vectors using the sample mean and covariance matrix of the market and asset returns to obtain bootstrapped statistics. ** and * denote significance at the 1% and 5% level.

Test Portfolio	Annualized Mean Excess Return(%)	Annualized Volatility (%)	Market β	Skewness	Coskewness $(x10^6)$
Panel A: 7 U.S. Treasury Portfolios Sorted on Maturity - 07/1961 to 12/2012					
1yr	1.03	1.59	0.40	1.36^{**}	0.25^{**}
2yr	1.28	2.81	0.78	0.94^{**}	0.42^{**}
5yr	1.94	5.30	1.62	0.15	0.67^{**}
7yr	2.42	6.44	1.98	0.26^{**}	0.84^{**}
10yr	2.20	7.67	2.30	0.25^{**}	0.93^{**}
20yr	2.69	10.21	3.02	0.41^{**}	1.51^{**}
30yr	2.52	11.39	3.28	0.49^{**}	1.54^{**}
Correlation with $E[r]$		0.93	0.95	-0.81	0.91
Panel B: 5 Currency Forward Portfolios Sorted on Forward Discount - 11/1983 to 12/2012					
Low	0.43	9.03	-0.01	0.31^{*}	14.28^{**}
2	0.99	9.13	0.06	-0.14	5.48^{**}
3	2.86	9.08	0.10	-0.24	1.46^{**}
4	4.46	9.39	0.14	-0.30^{*}	-1.89^{**}
High	5.78	11.27	0.18	-0.30^{*}	-8.00^{**}
Correlation with $E[r]$		0.79	0.97	-0.78	-0.95
Panel C: 5 Commodity Futures Portfolios Sorted on Basis - 01/1970 to 12/2012					
Low	0.40	21.46	0.15	0.70^{**}	-14.16^{**}
2	4.18	21.45	0.20	1.69^{**}	-18.71^{**}
3	6.56	21.60	0.19	0.26^{*}	-21.76^{**}
4	9.57	19.41	0.16	0.22^{*}	-9.54^{**}
High	12.89	23.24	0.09	0.52^{**}	-14.68^{**}
Correlation with $E[r]$		0.20	-0.61	-0.43	0.25

Table 2 Summary Statistics For Bonds, FX, and Commodities

I report sample annualized average excess returns, annualized volatility, market β, skewness, coskewness with the market of monthly excess returns. Panel A reports on seven maturity sorted U.S. Treasury constant maturity indices, panel B on five forward discount sorted portfolios of currency forwards, and panel C on five basis sorted portfolios of commodity futures. The market portfolio for U.S. Treasury indices is the value-weighted portfolio of all Treasury issues. The market portfolio for currency forward portfolios and commodity futures portfolios is the value-weighted U.S. equity portfolio. In each panel, I report the cross-sectional correlation of each measure with average excess returns. I compute bootstrap p-values for the null hypotheses of zero skewness and zero non-normal coskewness. I simulate 10,000 bivariate normal vectors using the sample mean and covariance matrix of the market and asset returns to obtain bootstrapped statistics. ** and * denote significance at the 1% and 5% level.

Test Portfolio	Annualized Mean Excess Return(%)	Annualized Volatility (%)	Market β	Skewness	Coskewness $(x10^6)$
Panel A: 25 U.S. Equity Portfolios Sorted on Size and Momentum - 07/1961 to 12/2012					
Small - Loser	−0.54	28.08	1.37	0.89**	−38.14**
Small - Momentum 2	7.39	20.58	1.05	0.11	−38.89**
Small - Momentum 3	10.60	18.97	0.99	−0.27**	−42.56**
Small - Momentum 4	12.01	19.15	1.00	−0.56**	−48.23**
Small - Winner	16.06	23.55	1.21	−0.43**	−58.53**
Size 2 - Loser	0.89	27.52	1.45	0.62**	−34.09**
Size 2 - Momentum 2	7.05	20.55	1.12	0.04	−33.53**
Size 2 - Momentum 3	9.14	18.38	1.03	−0.41**	−36.23**
Size 2 - Momentum 4	11.42	18.79	1.05	−0.59**	−42.99**
Size 2 - Winner	14.10	23.47	1.28	−0.40**	−54.26**
Size 3 - Loser	2.22	25.83	1.37	0.45**	−22.00**
Size 3 - Momentum 2	6.40	19.33	1.10	−0.04	−26.86**
Size 3 - Momentum 3	7.77	17.64	1.01	−0.34**	−31.77**
Size 3 - Momentum 4	8.64	17.39	1.00	−0.74**	−40.89**
Size 3 - Winner	13.66	21.86	1.21	−0.52**	−49.08**
Size 4 - Loser	1.46	25.44	1.34	0.40**	−17.18**
Size 4 - Momentum 2	6.13	19.35	1.11	−0.03	−23.93**
Size 4 - Momentum 3	7.14	17.01	1.00	−0.28**	−26.86**
Size 4 - Momentum 4	8.99	16.70	0.99	−0.51**	−30.06**
Size 4 - Winner	11.84	20.47	1.14	−0.54**	−46.49**
Large - Loser	1.04	23.70	1.24	0.20*	−12.65**
Large - Momentum 2	4.83	17.23	0.94	0.13	−8.57**
Large - Momentum 3	3.95	15.33	0.90	−0.30**	−24.61**
Large - Momentum 4	6.00	15.14	0.89	−0.19	−21.72**
Large - Winner	8.83	18.32	1.02	−0.42**	−32.78**
Correlation with $E[r]$		−0.33	−0.32	−0.83	−0.77
Panel B: 5 Developed Equity Market Portfolios Sorted on Momentum - 02/1973 to 12/2012					
Loser	0.71	22.73	1.05	1.17**	1.35**
2	4.42	19.12	0.97	−0.34**	−23.94**
3	9.01	17.32	0.96	−0.36**	−27.03**
4	9.51	17.86	0.93	−0.46**	−31.32**
Winner	6.91	20.80	1.02	−1.18**	−47.65**
Correlation with $E[r]$		−0.86	−0.80	−0.75	−0.74

Table 3 Summary Statistics For Momentum Sorted Equity Portfolios
I report sample annualized average excess returns, annualized volatility, market β, skewness, coskewness with the market of monthly excess returns. Panel A reports on 25 size and momentum sorted U.S. equity portfolios, and panel B on five momentum sorted value-weighted portfolios of 16 developed country equity markets. The market portfolio for U.S. equity portfolios is the CRSP value-weighted market portfolio. The market portfolio for international equity portfolios is the value-weighted portfolio with all country indices. In each panel, I report the cross-sectional correlation of each measure with average excess returns. I compute bootstrap p-values for the null hypotheses of zero skewness and zero non-normal coskewness. I simulate 10,000 bivariate normal vectors using the sample mean and covariance matrix of the market and asset returns to obtain bootstrapped statistics. ** and * denote significance at the 1% and 5% level.

Test Portfolio	Annualized Mean Excess Return(%)	Annualized Volatility (%)	Market β	Skewness	Coskewness $(x10^6)$
Panel A: 3 U.S. Treasury Portfolios Sorted on Momentum - 01/1962 to 12/2012					
Loser	1.44	7.26	1.78	0.59**	0.91**
2	2.05	6.34	1.93	0.41**	0.88**
Winner	2.58	7.69	2.01	0.25*	0.87**
Correlation with $E[r]$		0.28	0.99	−1.00	−0.96
Panel B: 5 Currency Forward Portfolios Sorted on Momentum - 11/1984 to 12/2012					
Loser	1.67	9.89	0.13	0.28*	−2.07**
2	3.95	10.08	0.09	−0.10	1.92**
3	3.99	9.69	0.07	−0.21	5.75**
4	4.46	9.57	0.06	−0.12	7.31**
Winner	4.72	9.49	0.13	−0.27*	−10.90**
Correlation with $E[r]$		−0.53	−0.39	−0.96	0.02
Panel C: 5 Commodity Futures Portfolios Sorted on Momentum - 01/1970 to 12/2012					
Loser	−1.15	22.19	0.21	0.80**	−18.11**
2	1.90	18.53	0.14	0.93**	−17.06**
3	6.86	19.26	0.12	0.88**	−10.20**
4	11.76	20.85	0.13	1.63**	−21.89**
Winner	15.37	28.08	0.19	0.73**	−19.69**
Correlation with $E[r]$		0.60	−0.20	0.28	−0.31

Table 4 Summary Statistics For Momentum Sorted Portfolios of Bonds, FX, and Commodities

I report sample annualized average excess returns, annualized volatility, market β, skewness, coskewness with the market of monthly excess returns. Panel A reports on three momentum sorted U.S. Treasury constant maturity indices, panel B on five momentum sorted portfolios of currency forwards, and panel C on five momentum portfolios of commodity futures. The market portfolio for U.S. Treasury indices is the value-weighted portfolio of all Treasury issues. The market portfolio for currency forward portfolios and commodity futures portfolios is the value-weighted U.S. equity portfolio. In each panel, I report the cross-sectional correlation of each measure with average excess returns. I compute bootstrap p-values for the null hypotheses of zero skewness and zero non-normal coskewness. I simulate 10,000 bivariate normal vectors using the sample mean and covariance matrix of the market and asset returns to obtain bootstrapped statistics. ** and * denote significance at the 1% and 5% level.

Risk Factor	Annualized Mean Excess Return(%)	Annualized Volatility (%)	Skewness	Correlation with $\tilde{r}_{SA}$	Correlation with r_{IA}
Panel A: 10 U.S. Equity Industry Portfolios - 07/1961 to 12/2012					
Market factor r_m	5.44**	15.63	−0.50**	−0.90	0.30
Systematic asymmetry factor $\tilde{r}_{SA}$	−6.30**	15.63	0.58**		−0.43
Idiosyncratic asymmetry factor r_{IA}	−0.21	15.63	0.26**		
Panel B: 16 Developed Equity Market Portfolios - 02/1973 to 12/2012					
Market factor r_m	5.15*	15.36	−0.46**	−0.62	0.07
Systematic asymmetry factor $\tilde{r}_{SA}$	−5.92**	15.36	1.06**		−0.08
Idiosyncratic asymmetry factor r_{IA}	2.20	15.36	0.72**		
Panel C: 7 U.S. Treasury Portfolios Sorted on Maturity - 07/1961 to 12/2012					
Market factor r_m	1.60**	3.14	0.31**	0.47	0.87
Systematic asymmetry factor $\tilde{r}_{SA}$	1.80**	3.14	2.15**		0.35
Idiosyncratic asymmetry factor r_{IA}	0.57	3.14	0.51**		
Panel D: 5 Currency Forward Portfolios Sorted on Forward Discount - 11/1983 to 12/2012					
Market factor r_m	6.98**	15.77	−0.83**	−0.30	−0.07
Systematic asymmetry factor $\tilde{r}_{SA}$	−8.01**	15.77	0.86**		0.23
Idiosyncratic asymmetry factor r_{IA}	1.95	15.77	0.52**		
Panel E: 5 Commodity Futures Portfolios Sorted on Basis - 01/1970 to 12/2012					
Market factor r_m	5.63*	16.16	−0.53**	−0.17	−0.01
Systematic asymmetry factor $\tilde{r}_{SA}$	−5.58*	16.16	−0.20		0.03
Idiosyncratic asymmetry factor r_{IA}	−0.46	16.16	0.75**		

Table 5 Summary Statistics for Risk Factors

I report for all asset classes sample annualized average excess returns, annualized volatility, skewness, and cross-correlations for the value-weighted market portfolio, the systematic asymmetry factor, and the idiosyncratic asymmetry factor constructed in Section 3.2. Asymmetry factors are constructed based on test portfolios sorted by asset class specific variables, and scaled such that their realized (ex post) volatility is equal to the market portfolio sample volatility. The market portfolio for U.S. equity, currency forward, and commodity futures portfolios is the CRSP value-weighted market portfolio. The market portfolio for international equity portfolios is the value-weighted portfolio with all country indices. The market portfolio for U.S. Treasury indices is the value-weighted portfolio of all Treasury issues. I test for the null hypothesis of zero average excess returns. I compute bootstrap p-values for the null hypotheses of zero skewness. I simulate 10,000 normally distributed vectors using the sample mean and covariance matrix of the market and factors returns to obtain bootstrapped statistics. ** and * denote significance at the 1% and 5% level.

Risk Factor	Annualized Mean Excess Return(%)	Annualized Volatility (%)	Skewness	Correlation with $\tilde{r}_{SA}$	Correlation with r_{IA}
Panel A: 25 U.S. Equity Portfolios Sorted on Size and Momentum - 07/1961 to 12/2012					
Market factor r_m	5.44**	15.63	−0.50**	−0.61	0.34
Systematic asymmetry factor $\tilde{r}_{SA}$	−8.22**	15.63	1.23**		−0.25
Idiosyncratic asymmetry factor r_{IA}	−2.24	15.63	1.46**		
Panel B: 5 Developed Equity Market Portfolios Sorted on Momentum - 02/1973 to 12/2012					
Market factor r_m	5.15*	15.36	−0.46**	−0.72	0.06
Systematic asymmetry factor $\tilde{r}_{SA}$	−8.22**	15.36	0.92**		0.52
Idiosyncratic asymmetry factor r_{IA}	−5.16*	15.36	0.30**		
Panel C: 3 U.S. Treasury Portfolios Sorted on Momentum - 01/1962 to 12/2012					
Market factor r_m	1.61**	3.16	0.30**	0.95	−0.05
Systematic asymmetry factor $\tilde{r}_{SA}$	0.93*	3.16	0.39**		0.07
Idiosyncratic asymmetry factor r_{IA}	−0.34	3.16	0.26**		
Panel D: 5 Currency Forward Portfolios Sorted on Momentum - 11/1984 to 12/2012					
Market factor r_m	7.38**	15.83	−0.89**	−0.26	0.01
Systematic asymmetry factor $\tilde{r}_{SA}$	−0.96	15.83	0.42**		0.18
Idiosyncratic asymmetry factor r_{IA}	−4.45	15.83	0.55**		
Panel E: 5 Commodity Futures Portfolios Sorted on Momentum - 01/1970 to 12/2012					
Market factor r_m	5.63*	16.16	−0.53**	−0.16	−0.03
Systematic asymmetry factor $\tilde{r}_{SA}$	−6.82**	16.16	−0.60**		0.16
Idiosyncratic asymmetry factor r_{IA}	7.03**	16.16	0.65**		

Table 6 Summary Statistics for Risk Factors Based on Momentum Sorted Portfolios
I report for all asset classes sample annualized average excess returns, annualized volatility, skewness, and cross-correlations for the value-weighted market portfolio, the systematic asymmetry factor, and the idiosyncratic asymmetry factor constructed in Section 3.2. Asymmetry factors are constructed based on test portfolios sorted by momentum, and scaled such that their realized (ex post) volatility is equal to the market portfolio sample volatility. The market portfolio for U.S. equity, currency forward, and commodity futures portfolios is the CRSP value-weighted market portfolio. The market portfolio for international equity portfolios is the value-weighted portfolio with all country indices. The market portfolio for U.S. Treasury indices is the value-weighted portfolio of all Treasury issues. I test for the null hypothesis of zero average excess returns. I compute bootstrap p-values for the null hypotheses of zero skewness. I simulate 10,000 normally distributed vectors using the sample mean and covariance matrix of the market and factors returns to obtain bootstrapped statistics. ** and * denote significance at the 1% and 5% level.

Model	GRS Statistic	p-value (%)	χ^2 Statistic	p-value (%)	Average $|\alpha|$ (bps)	Average σ_α (bps)	Average R^2
Panel A: 10 U.S. Equity Industry Portfolios - 07/1961 to 12/2012							
CAPM	2.13	2.03	20.96	2.13	12.79	0.119	0.66
FF3	4.05	0.00	40.50	0.00	17.23	0.113	0.70
FF3 + Momentum	3.64	0.01	34.75	0.01	15.33	0.115	0.70
A-CAPM	1.91	4.10	18.69	4.44	13.21	0.114	0.69
CAPM + r_{IA}	2.05	2.64	20.97	2.13	10.78	0.108	0.72
GA-CAPM	1.67	8.44	17.11	7.19	10.36	0.102	0.75
Panel B: 16 Developed Equity Market Portfolios - 02/1973 to 12/2012							
CAPM	0.65	84.40	10.09	86.22	20.32	0.221	0.48
FF3	0.76	72.61	12.22	72.88	12.68	0.222	0.49
FF3 + Momentum	0.94	52.50	13.24	65.54	14.55	0.227	0.49
A-CAPM	0.52	93.50	9.17	90.61	13.17	0.211	0.53
CAPM + r_{IA}	0.61	87.43	9.93	87.00	16.40	0.207	0.54
GA-CAPM	0.49	95.05	8.80	92.15	10.48	0.196	0.59
Panel C: 7 U.S. Treasury Portfolios Sorted on Maturity - 07/1961 to 12/2012							
CAPM	10.72	0.00	90.26	0.00	9.69	0.028	0.84
CAPM + TERM	7.61	0.00	58.25	0.00	5.07	0.023	0.89
A-CAPM	9.58	0.00	79.23	0.00	9.61	0.027	0.86
CAPM + r_{IA}	8.43	0.00	68.91	0.00	4.93	0.019	0.92
GA-CAPM	7.72	0.00	58.16	0.00	5.58	0.017	0.94
Panel D: 5 Currency Forward Portfolios Sorted on Forward Discount - 11/1983 to 12/2012							
CAPM	1.95	8.48	8.40	13.55	18.75	0.147	0.04
CAPM + HML$_{FX}$	0.55	74.08	2.62	75.88	12.51	0.139	0.15
A-CAPM	1.07	37.69	5.06	40.87	20.30	0.140	0.14
CAPM + r_{IA}	1.81	11.03	7.08	21.45	19.04	0.143	0.09
GA-CAPM	0.72	60.78	2.87	71.98	20.99	0.136	0.19
Panel E: 5 Commodity Futures Portfolios Sorted on Basis - 01/1970 to 12/2012							
CAPM	3.37	0.53	13.53	1.89	50.09	0.272	0.03
CAPM + HML$_C$	1.37	23.50	5.12	40.13	47.90	0.247	0.20
A-CAPM	2.64	2.26	12.06	3.40	34.90	0.204	0.44
CAPM + r_{IA}	3.36	0.54	13.51	1.90	50.05	0.256	0.14
GA-CAPM	2.63	2.30	12.03	3.44	34.35	0.172	0.55

Table 7 Time Series Regressions
For all asset classes, I report summary statistics for time series regressions of test portfolio excess returns on a constant and a set of factors (Equation (19)). The market portfolio for U.S. equity, currency forward, and commodity futures portfolios is the CRSP value-weighted market portfolio. The market portfolio for international equity portfolios is the value-weighted portfolio with all country indices. The market portfolio for U.S. Treasury indices is the value-weighted portfolio of all Treasury issues. In each panel, I report the Gibbons et al. (1989) (GRS) test statistics and the χ^2 test statistics for the null hypothesis of zero pricing errors as well as their p-values, the average absolute pricing error, the average standard error of the intercept coefficients, and the average of time series regression R^2s. CAPM includes only the market portfolio r_m as a factor. FF3 adds the small-minus-big and high-minus-low book-to-market factors, and FF3+Momentum adds the winner-minus-loser factor to FF3. TERM is the difference in monthly returns between the 10-year and 1-year Treasury bond. HML$_{FX}$ (HML$_C$) is a rank-based factor which is long high forward discount (basis) and short low forward discount (basis) quintile portfolios of currency forwards (commodity futures). A-CAPM refers to the model with r_m and the systematic asymmetry risk factor $\tilde{r}_{SA}$, CAPM+r_{IA} uses r_m and the idiosyncratic asymmetry risk factor r_{IA}. Finally, GA-CAPM refers to the model with r_m, $\tilde{r}_{SA}$, and r_{IA}.

Model	GRS Statistic	p-value (%)	χ^2 Statistic	p-value (%)	Average $\|\alpha\|$ (bps)	Average σ_α (bps)	Average R^2
Panel A: 25 U.S. Equity Portfolios Sorted on Size and Momentum - 07/1961 to 12/2012							
CAPM	5.30	0.00	126.54	0.00	33.93	0.123	0.74
FF3	4.94	0.00	154.63	0.00	33.32	0.095	0.85
FF3 + Momentum	3.70	0.00	95.65	0.00	12.83	0.072	0.91
A-CAPM	4.92	0.00	113.70	0.00	24.72	0.115	0.78
CAPM + r_{IA}	5.10	0.00	104.96	0.00	34.99	0.095	0.84
GA-CAPM	4.71	0.00	99.83	0.00	25.73	0.085	0.87
Panel B: 5 Developed Equity Market Portfolios Sorted on Momentum - 02/1973 to 12/2012							
CAPM	3.71	0.27	21.56	0.06	26.21	0.163	0.61
FF3	3.37	0.53	20.61	0.10	26.75	0.165	0.62
FF3 + Momentum	2.58	2.56	15.87	0.72	21.06	0.168	0.62
A-CAPM	2.28	4.61	12.86	2.47	17.12	0.136	0.73
CAPM + r_{IA}	2.68	2.11	15.87	0.72	14.91	0.137	0.72
GA-CAPM	2.27	4.68	12.80	2.53	16.78	0.118	0.79
Panel C: 3 U.S. Treasury Portfolios Sorted on Momentum - 01/1962 to 12/2012							
CAPM	13.83	0.00	53.60	0.00	8.68	0.042	0.74
CAPM + TERM	7.26	0.01	20.12	0.02	5.08	0.041	0.75
A-CAPM	8.08	0.00	27.29	0.00	9.88	0.030	0.86
CAPM + r_{IA}	13.68	0.00	53.12	0.00	8.78	0.026	0.88
GA-CAPM	7.75	0.00	24.57	0.00	8.25	0.017	0.96
Panel D: 5 Currency Forward Portfolios Sorted on Momentum - 11/1984 to 12/2012							
CAPM	1.51	18.54	7.13	21.14	25.48	0.153	0.04
CAPM + HML_{FX}	1.38	23.11	5.97	30.87	19.83	0.152	0.07
A-CAPM	1.48	19.45	7.07	21.55	25.59	0.149	0.08
CAPM + r_{IA}	1.06	38.03	4.53	47.56	25.71	0.144	0.15
GA-CAPM	0.98	42.84	4.05	54.28	25.92	0.141	0.19
Panel E: 5 Commodity Futures Portfolios Sorted on Momentum - 01/1970 to 12/2012							
CAPM	4.60	0.04	21.52	0.06	58.27	0.277	0.03
CAPM + HML_C	3.12	0.88	11.95	3.55	50.55	0.274	0.06
A-CAPM	3.40	0.49	17.67	0.34	46.79	0.209	0.44
CAPM + r_{IA}	2.87	1.44	13.45	1.95	48.98	0.258	0.17
GA-CAPM	1.21	30.12	6.49	26.12	25.00	0.180	0.60

Table 8 Time Series Regressions For Momentum Sorted Portfolios

For all asset classes, I report summary statistics for time series regressions of momentum sorted test portfolio excess returns on a constant and a set of factors (Equation (19)). The market portfolio for U.S. equity, currency forward, and commodity futures portfolios is the CRSP value-weighted market portfolio. The market portfolio for international equity portfolios is the value-weighted portfolio with all country indices. The market portfolio for U.S. Treasury indices is the value-weighted portfolio of all Treasury issues. In each panel, I report the Gibbons et al. (1989) (GRS) test statistics and the χ^2 test statistics for the null hypothesis of zero pricing errors as well as their p-values, the average absolute pricing error, the average standard error of the intercept coefficients, and the average of time series regression R^2s. CAPM includes only the market portfolio r_m as a factor. FF3 adds the small-minus-big and high-minus-low book-to-market factors, and FF3+Momentum adds the winner-minus-loser factor to FF3. TERM is the difference in monthly returns between the 10-year and 1-year Treasury bond. HML_{FX} (HML_C) is a rank-based factor which is long high forward discount (basis) and short low forward discount (basis) quintile portfolios of currency forwards (commodity futures). A-CAPM refers to the model with r_m and the systematic asymmetry risk factor $\tilde{r}_{SA}$, CAPM+r_{IA} uses r_m and the idiosyncratic asymmetry risk factor r_{IA}. Finally, GA-CAPM refers to the model with r_m, $\tilde{r}_{SA}$, and r_{IA}.